and all y_{j2}, i.e., y_{12} and y_{22} are negative, so this LPP admits unbounded solution.

<u>Note</u>: If $Z_j - c_j < 0$ for some j and all components of y_j is negative or zero, then the LPP has unbounded solution.

Unbounded means, for maximization problem, you can increase Z as much as possible and for minimization problem, you can decrease Z as ~~most as~~ much as possible.

<u>Exercise 2</u> Solve using simplex method, the following LPP:

$$\text{Minimize } Z = x_1 - 3x_2 + 2x_3$$

$$\text{subject to } \begin{aligned} 3x_1 - x_2 + 2x_3 &\le 7 \\ -2x_1 + 4x_2 &\le 12 \\ -4x_1 + 3x_2 + 8x_3 &\le 10 \end{aligned}$$

$$x_1, x_2, x_3 \geq 0$$

<u>Example 3</u> Use Charnes' Big-M method (or penalty method) to solve the following LPP:

$$\text{Maximize } Z = 3x_1 - x_2$$

$$\text{subject to } \begin{aligned} 2x_1 + x_2 &\geq 2 \\ x_1 + 3x_2 &\le 3 \\ x_2 &\le 4 \end{aligned}$$

$$x_1, x_2 \geq 0$$

Solution: Using slack variables x_4, x_5 and surplus variable x_3 and ~~arti~~ artificial variable x_6, the problem can be put in the standard form as follows

$$\text{Maximize } Z = 3x_1 - x_2 + 0.x_3 + 0.x_4 + 0.x_5 - M.x_6 \quad (M \text{ is a large positive real number})$$

Subject to

$$\begin{aligned} 2x_1 + x_2 - x_3 \qquad\qquad + x_6 &= 2 \\ x_1 + 3x_2 \qquad + x_4 \qquad\qquad &= 3 \\ x_2 \qquad\qquad + x_5 \qquad &= 4 \end{aligned}$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

So, we form the simplex tables as follows:

		c_j		3	-1	0	0	0	-M	Minimum
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5	y_6	ratio
-M	a_6	x_6	2	$\boxed{2}$	1	-1	0	0	1	$2/2 = 1$ ←
0	a_4	x_4	3	1	3	0	1	0	0	$3/1 = 3$
0	a_5	x_5	4	0	1	0	0	1	0	
$Z_j - C_j$				$-2M-3$ ↑	$-M+1$	M	0	0	0 ↓	Minimum ratio
3	a_1	x_1	1	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0		
0	a_4	x_4	2	0	$\frac{5}{2}$	$\boxed{\frac{1}{2}}$	1	0		$\frac{2}{1/2} = 4$ ←
0	a_5	x_5	4	0	1	0	0	1		
$Z_j - C_j$				0	$5/2$	$-3/2$ ↑	0	0		
3	a_1	x_1	3	1	3	0	1	0		
0	a_3	x_3	4	0	5	1	2	0		
0	a_5	x_5	4	0	1	0	0	1		
$Z_j - C_j$				0	10	0	3	0		

In Table 1, a_1 is the entering vector, a_6 is the departing vector.

In table 2 we need not calculate the column corresponding to artificial variable as it is not for a basic variable now. ~~Hence the entering vector is~~. As $y_{13} = -\frac{1}{2} < 0$ and $y_{33} = 0$, so we get only one ratio. Hence the entering vector is a_3 and the departing vector is a_4.

Since all $z_j - c_j \geq 0$ and artificial variable is not in the optimal solution, so the ~~total~~ optimal solution is

$$x_1 = 3, \quad x_2 = 0 \quad \text{and} \quad Z_{max} = 3 \times 3 - 0 = 9$$

Note: Sometimes, in the problem nothing is written about using ~~changes~~ Charne's Big-M method, you have to use it when needed. For this, Do it yourself, the next exercises.

Exercise 3 Solve the following LPP by simplex method:

$$\text{Maximize } Z = 4x_1 + 2x_2$$
$$\text{subject } 2x_1 + x_2 \leq 4$$
$$5x_1 + 3x_2 \geq 15$$
$$\text{and } x_1, x_2 \geq 0$$

Exercise 4 Solve the following LPP by Big-M method:

$$\text{Minimize } Z = 2x_1 + 3x_2$$
$$\text{subject to } 2x_1 + 7x_2 \geq 22$$
$$x_1 + x_2 \geq 6$$
$$5x_1 + x_2 \geq 10$$
$$x_1, x_2 \geq 0$$

Concept of Duality

To every LPP there corresponds another LPP. If the first LPP is called primal then the later is called the dual problem and vice-versa. It will be seen later that every primal problem has a unique dual problem. If the primal is of maximization, then the dual is of minimization problem and vice-versa. That is, if some wants to maximize profit there is correspondingly some who wants to minimize its cost and similarly if someone wants to minimize its cost then corresponding there is someone who wants to maximize its profit. Also we will see that while solving and LPP for the primal, we are at the same time solving its dual problem as well.

To make the concept more clear, let us consider the following problem:

Types of food	Units of vitamins present/gm.		Cost price per gm of the foods
	A	B	
X	5	6	Rs. 1
Y	8	10	Rs. 2
Daily minimum requirement of vitamins in units	30	40	

Let x_1 gms of the food X and x_2 gm of foods Y be purchased and the problem is to minimize the cost.

Now the LPP becomes

Minimize $z = x_1 + 2x_2$

subject to $5x_1 + 8x_2 \geq 80$

$6x_1 + 10x_2 \geq 100$

and $x_1, x_2 \geq 0$

If we consider this as a primal problem, consider the

Corresponding problem as follows: Let a dealer ~~sates~~ sells vitamin A and vitamin B required for two types of foods, X and Y. His problem is to fix the cost per unit of vitamin A and vitamin B in such a way that the ~~prices~~ ~~for~~ prices per gm of food X and Y do not exceed the amount mentioned above. His problem is also to get a maximum profit in selling the vitamins.

Let ~~v~~ v_1 and v_2 be the price per unit of vitamin A and vitamin B respectively. Therefore his problem is to

$$\text{Maximize } W = 30v_1 + 40v_2$$

$$\text{subject to } 5v_1 + 6v_2 \leq 1$$

$$8v_1 + 10v_2 \leq 2$$

$$v_1, v_2 \geq 0$$

This problem is called the dual of the first problem. Similarly if you consider the 2nd problem as the primal problem then the dual would be the first problem.

<u>Formulation of primal-dual problems</u>

Let the standard primal problem be

$$\text{Maximize } Z = cx$$

$$\text{subject to } Ax \leq b$$

$$x \geq 0$$

Then the dual problem is

$$\text{Minimize } W = b^t v$$

$$\text{subject to } A^t v \geq c^t, \quad v \geq 0$$