We shall prove that $x_{ij} = \frac{a_i b_j}{M}$, for all i and j, will be a feasible solution of the TP.

Now $\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} \frac{a_i b_j}{M} = \frac{a_i}{M} \sum_{j=1}^{n} b_j = \frac{a_i}{M} \cdot M = a_i$, $i = 1, 2, \ldots m$

and also $\sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} \frac{a_i b_j}{M} = \frac{b_j}{M} \sum_{i=1}^{m} a_i = \frac{b_j}{M} \cdot M = b_j$, $j = 1, 2, \ldots, n$

So, this $x_{ij} = \frac{a_i b_j}{M}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ satisfies all the constraints of TP. Also $x_{ij} \geq 0$ for all i and j as $a_i > 0$ and $b_j > 0$ for all i and j

Thus $x_{ij} = \frac{a_i b_j}{M}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ is a feasible solution of the TP.

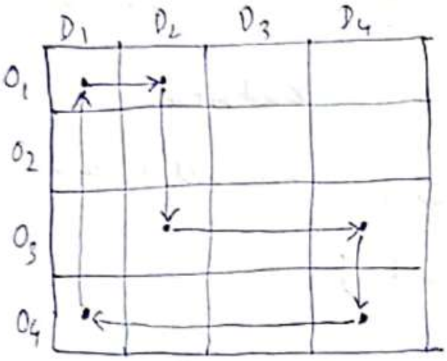
Theorem 3 The solution of a ~~bcan~~ balanced transportation problem is never unbounded.

Proof: From theorem 2, there exists a feasible solution to the TP. So, $\exists$ a basic feasible solution to the TP. As all c_{ij} and x_{ij} are finite quantities, then Z is a bounded function. Hence the problem has an optimal value and there exists a basic feasible solution which will be an optimal solution to ~~the~~ the problem.

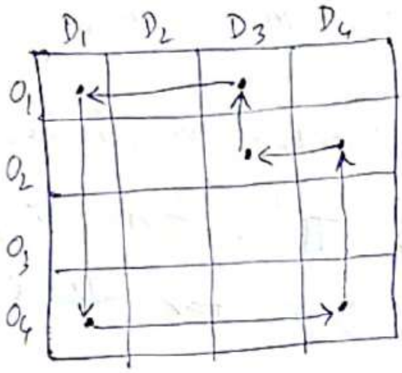
Loops in a Transportation table

In a transportation table, an ordered set of four or more cells are said to form a Loop if and only if two consecutive cells in the ordered set lie either in the same row or in the same column and if the first and the last cell of the set also lie either in the same row or in the same column.

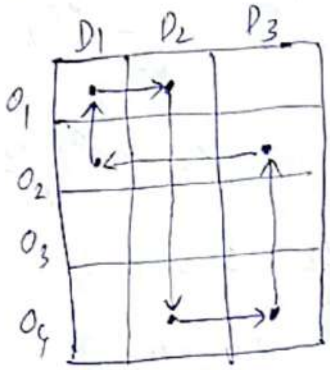
Some loops are shown in the following figures:



Loop L_1



Loop L_2



loop L_3

Loop $L_1 = \{(1,1),(1,2),(3,2),(3,4),(4,4),(4,1)\}$

Loop $L_2 = \{(1,1),(4,1),(4,4),(2,4),(2,3),(1,3)\}$

Loop $L_3 = \{(1,1),(1,2),(4,2),(4,3),(2,3),(2,1)\}$

<u>Note 1</u> If a set of cells in a TP contains a loop then we can prove that corresponding columns are linearly dependent. So, if we choose cells which do not contain loop, then, their corresponding columns are linearly independent.

<u>Note 2</u> To find a basic feasible solution, we choose $(m+n-1)$ cells in such a way that those cells do not contain any loop. Then the ~~basic~~ variables related to the corresponding cells should be basic variable. So, if we can find out

$x_{ij} \geq 0$ for $(m+n-1)$ cells which do not contain any loop, then we get ~~the~~ a basic feasible solution where those $(m+n-1)$ x_{ij}'s are basic components and all the nonbasic components are zero. This is the technique to find an initial basic feasible solution to the TP.

Methods for finding initial basic feasible solution

The methods for finding initial basic feasible solution to be discussed here are

(1) North-West Corner Rule
(2) Row minima method
(3) Column minima method
(4) ~~Mita~~ Matrix minima method
(5) Vogel's Approximation method (VAM)

Now the above methods are discussed with examples:.

(1) North-West Corner Rule

Step 1 Allocate in the cell at the North-West corner, i.e., at (1, 1) cell of the table, the maximum amount allowable so that either the capacity of first row is exhausted or the demand of the first column is satisfied.

Thus $x_{11} = \min(a_1, b_1)$

Step 2 (i) If ~~a~~ $b_1 > a_1$, we cross out the first row and adjust the associated amounts of supply and demand by subtracting the allocated amount. Then move down vertically to the ~~next row~~ second row and allocate in the (2,1) cell an amount x_{21}, where $x_{21} = \min(a_2, b_1 - x_{11})$.

(ii) If $a_1 > b_1$, we cross out the first column and adjust the associated amounts of supply and demand by subtracting the allocated amount, and then we move right horizontally to the second column and allocate in the ~~cell~~ (1,2) cell an amount x_{12} where $x_{12} = \min(a_1 - x_{11}, b_2)$.

(iii) If $a_1 = b_1$, there is a tie, we cross out both the rows and columns and go to the cell (2,2) and take either $x_{12} = 0$ or $x_{21} = 0$

Step 3 Now allocate in the same way as before to ~~to~~ the new matrix and move forward to get a basic feasible solution.

For case (iii) of Step 2, we get a degenerate basic feasible solution i.e where some basic component is zero.

~~Example~~

Note: we always make a TP balanced one if it is not to get always a feasible solution, so that we can get at least one basic feasible solution.

If $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$, then if $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$ then we make the problem a balanced one by introducing a dummy ~~of~~ or fictitious ~~origin~~ destination to the transportation table and the requirement at this dummy destination will be assumed to be $\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$ and the transportation cost from any source to the dummy destination is taken as zero.

If $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$ then ~~we~~ to make the problem balanced we take a dummy source or origin and