So, an initial basic feasible solution by Row minima method is $x_{11}=3$, $x_{14}=1$, $x_{24}=2$, $x_{25}=6$, $x_{32}=3$, $x_{33}=4$, $x_{34}=2$

It is a non-degenerate basic feasible solution as the number component is $7=(5+3-1)$

The cost corresponding to this solution is

$$= 3\times2 + 1\times3 + 2\times2 + 6\times1 + 3\times9 + 4\times4 + 2\times8$$

$$= 6+3+4+6+27+16+16 = 78$$

③ Column minima method:

This method is exactly same as row minima method, the only difference is that instead of row we start with columns.

So, here we did not given any example. Try this method with previous example as exercise.

(4) Matrix method (Least cost entry method)

This method finds a better starting solution. In this method we first find out the cell with minimum cost in the cost matrix and allocate in that cell the maximum allowable amount. We then cross the satisfied row or column or both and adjust amount of supply and demand accordingly. Then we repeat the same process with the new matrix.

Example 3 Apply matrix minima method to find an initial basic feasible solution to the ~~transportable~~ transportation problem of Example 1.

Solution :

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	2	11	10	3	7	4
O_2	$\boxed{3}$ 1	4	7	2	1	8
O_3	3	9	4	8	12	9
b_j	3	3	4	5	6	

In the cost matrix, cell (2, 1) and (2, 5) have the minimum cost. So, we select any one of these. Let us select (2,1) cell allocate

$x_{21} = \min(a_2, b_1) = \min(8, 3) = 3$

Demand of the first column is fulfilled. So, we cross out the first column. We go to the next table with adjusted requirements and demands.

	D_2	D_3	D_4	D_5	a_i
O_1	11	10	3	7	4
O_2	4	7	2	$\boxed{5}$ 1	5 (=8 − 3)
O_3	9	4	8	12	9
b_j	3	4	5	6	

Consider as before, $x_{25} = 5$. We cross out 2nd row.

The next table is

	D_2	D_3	D_4	D_5	
O_1	11	10	$\boxed{4}$ 3	7	4
O_3	9	4	8	12	9
	3	4	5	1 (=6 − 5)	

Similarly, $x_{14} = 4$, we cross out 1st row.

The next table is

	D_2	D_3	D_4	D_5	
O_3	9	$\boxed{4}$ 4	8	12	9
	3	4	1 (= 5 − 4)	1	

~~and finally~~ next, $x_{33} = 4$, cross out 3rd column

The next table is

	D_2	D_4	D_5	
O_3	9	$\boxed{1}$ 8	12	5 (=9-4)
	3	1	1	

Next, $x_{34} = 1$, we cross out column D_4. The next table is

	D_2	D_5	
O_3	$\boxed{3}$ 9	12	4 (=5-1)
	3	1	

Next $x_{32} = 3$, we cross out column D_2. Next table

is

	D_5	
O_3	$\boxed{1}$ 12	1 (=4-3)
	1	

So, last variable is $x_{35} = 1$.

So, a basic feasible solution by matrix minima method is

$x_{21} = 3$, $x_{25} = 5$, $x_{34} = 4$, $x_{33} = 4$, $x_{34} = 1$, $x_{32} = 3$, $x_{35} = 1$

The solution can be shown in a single table as follows

	D_1	D_2	D_3	D_4	D_5	
O_1	2	11	10	$\boxed{4}$ 3	7	4
O_2	$\boxed{3}$ 1	4	7	2	$\boxed{5}$ 1	8
O_3	3	$\boxed{3}$ 9	$\boxed{4}$ 4	$\boxed{1}$ 8	$\boxed{1}$ 12	9
	3	3	4	5	6	

It is a non-degenerate solution as the non-zero component is $7 = (5+3) - 1$

and the corresponding cost $= 4\times3 + 3\times1 + 5\times1 + 3\times9 + 4\times4 + 1\times8 + 1\times12$

$= 83$

(5) Vogel's approximation (VAM) or Unit Penalty method.

VAM is an improved version of Matrix minima method and produces generally a better starting solution in respect of cost minimization.

The steps to be followed in this method are:

Step 1 Find out for each row and each column the smallest unit cost and the next smallest unit cost (If there are two cell in a row or in a column with same smallest unit cost, then the next smallest unit cost would be the same). Then determine a penalty measure for each row and each column by taking the difference of these two and display these differences in parenthesis in the transportation table by the side of the availabilities in case of rows and below the requirements in the case of columns.

Step 2 Identify that row or column with largest penalty (if there is tie, take any one of them) and in this selected row or column, allocate the maximum allowable amount to the least cost cell, i.e., if this cell is (i, j), then $x_{ij} = \min(a_i, b_j)$. Adjust the supply and demand and cross out the satisfied row or column. If a row and a column be satisfied simultaneously, then cross out both of them (In this case, we always get a degenerate basic feasible solution and we have to choose another zero component so that loop is not formed) and the supply and demands are adjusted accordingly. Then we go to the next table and repeat the same process. In this way, we get a basic feasible solution.

Example 4 Use Vogel's Approximation Method to obtain an initial basic feasible solution of the same