Ex. 7 Without using truth table find the full dnf of the following statement forms:

(i) $\neg A \wedge (B \Leftrightarrow A)$

(ii) $((\neg A \wedge B) \vee \neg(A \wedge C)) \wedge \neg(A \vee (B \wedge C))$

(iii) $\neg((A \wedge B) \vee (A \wedge \neg C)) \vee \neg B$

Solution : (i)

$\neg A \wedge (B \Leftrightarrow A) \equiv \neg A \wedge ((B \wedge A) \vee (\neg B \wedge \neg A))$ [$\because A \Leftrightarrow B \equiv (A \wedge B) \vee (\neg A \wedge \neg B)$]

$\equiv (\neg A \wedge (B \wedge A)) \vee (\neg A \wedge (\neg B \wedge \neg A))$ [Distributive law]

$\equiv (\neg A \wedge (A \wedge B)) \vee (\neg A \wedge (\neg A \wedge \neg B))$ [Commutative Law]

$\equiv ((\neg A \wedge A) \wedge B) \vee ((\neg A \wedge \neg A) \wedge \neg B)$ [Associative Law]

$\equiv (F \wedge B) \vee (\neg A \wedge \neg B)$ [$\because \neg A \wedge A \equiv F$ and $A \wedge A \equiv A$]

$\equiv F \vee (\neg A \wedge \neg B)$

$\equiv (\neg A \wedge \neg B)$ [$\because F \vee A = A$]

This is the required full dnf

(ii) $((\neg A \wedge B) \vee \neg(A \wedge C)) \wedge \neg(A \vee (B \wedge C))$

$\equiv ((\neg A \wedge B) \vee (\neg A \vee \neg C)) \wedge (\neg A \wedge \neg(B \wedge C))$ [De Morgan's law]

$\equiv (((\neg A \wedge B) \vee \neg A) \vee \neg C) \wedge (\neg A \wedge \neg(B \wedge C))$ [Associative Law]

$\equiv (\neg A \vee \neg C) \wedge (\neg A \wedge \neg(B \wedge C))$ [$\because (A \wedge B) \vee A = A$ (Absorption law)]

$\equiv ((\neg A \vee \neg C) \wedge \neg A) \wedge (\neg B \vee \neg C)$ [Associative law and De Morgan's law]

$\equiv ((\neg C \vee \neg A) \wedge \neg A) \wedge (\neg B \vee \neg C)$ [Commutative Law]

$\equiv (\neg C \vee (\neg A \wedge \neg A)) \wedge (\neg B \vee \neg C)$ [Associative Law]

$\equiv (\neg C \vee \neg A) \wedge (\neg B \vee \neg C)$ [Idempotent Law]

$\equiv (\neg A \vee \neg C) \wedge (\neg B \vee \neg C)$ [Commutative Law]

$\equiv ((\neg A \vee \neg C) \wedge \neg B) \vee ((\neg A \vee \neg C) \wedge \neg C)$ (Distributive Law)

$\equiv (\neg A \wedge \neg B) \vee (\neg C \wedge \neg B) \vee (\neg A \wedge \neg C) \vee (\neg C \wedge \neg C)$ (Distributive law)

$\equiv$

$\equiv (\neg A\wedge\neg B)\vee(\neg B\wedge\neg C)\vee(\neg A\wedge\neg C)\vee(\neg C)$ (Commutative and Idempotent laws)

$\equiv ((\neg A\vee\neg C)\wedge\neg B)\vee(\neg C)$ (Absorption Law)

$\equiv ((\neg A\wedge\neg B)\vee(\neg C\wedge\neg B))\vee(\neg C)$ (Distributive Law)

$\equiv (\neg A\wedge\neg B)\vee((\neg C\wedge\neg B)\vee\neg C)$ (Associative law)

$\equiv (\neg A\wedge\neg B)\vee\neg C$ (Absorption law)

$\equiv ((\neg A\wedge\neg B)\wedge(C\vee\neg C))\vee\neg C$ $(C\vee\neg C\equiv T)$

$\equiv (\neg A\wedge\neg B\wedge C)\vee(\neg A\wedge\neg B\wedge\neg C)\vee((B\vee\neg B)\wedge\neg C)$

$\equiv (\neg A\wedge\neg B\wedge C)\vee(\neg A\wedge\neg B\wedge\neg C)\vee(B\wedge\neg C)\vee(\neg B\wedge\neg C)$

$\equiv (\neg A\wedge\neg B\wedge C)\vee(\neg A\wedge\neg B\wedge\neg C)\vee((A\vee\neg A)\wedge(B\wedge\neg C))\vee((A\vee\neg A)\wedge(\neg B\wedge\neg C))$

$\equiv (\neg A\wedge\neg B\wedge C)\vee(\neg A\wedge\neg B\wedge\neg C)\vee(A\wedge B\wedge\neg C)\vee(\neg A\wedge B\wedge\neg C)\vee(A\wedge\neg B\wedge\neg C)\vee(\neg A\wedge\neg B\wedge\neg C)$

$\equiv (\neg A\wedge\neg B\wedge C)\vee(\neg A\wedge\neg B\wedge\neg C)\vee(A\wedge B\wedge\neg C)\vee(\neg A\wedge B\wedge\neg C)\vee(A\wedge\neg B\wedge\neg C)$

$\equiv ((\neg A\vee\neg C)\wedge\neg A)\wedge(\neg B\vee\neg C)$ (Associative & De Morgan's Laws)

(Absorption Law)

$\equiv \neg A\wedge(\neg B\vee\neg C)$

(Distributive Law)

$\equiv (\neg A\wedge\neg B)\vee(\neg A\wedge\neg C)$

$\equiv ((\neg A\wedge\neg B)\wedge(C\vee\neg C))\vee((\neg A\wedge\neg C)\wedge(B\vee\neg B))$ [$\because A\vee\neg A\equiv T$ and $A\wedge T\equiv A$]

$\equiv (\neg A\wedge\neg B\wedge C)\vee(\neg A\wedge\neg B\wedge\neg C)\vee(\neg A\wedge B\wedge\neg C)\vee(\neg A\wedge\neg B\wedge\neg C)$ [using Distributive and Commutative Law]

This is required full dnf

(iii) Do it yourself.

Ex 8 Without using the truth table find the full cnf of the following statement forms:

(i) $(A\wedge\neg B)\vee(B\wedge\neg C)\vee(C\wedge\neg A)$

(ii) $\neg((A\wedge\neg B)\vee(A\wedge C))\vee\neg A$

(iii) $(A\wedge B)\vee(\neg A\wedge C)$

Solution (i)

$(A \wedge \neg B) \vee (B \wedge \neg C) \vee (C \wedge \neg A)$

$\equiv ((A \wedge \neg B) \vee (B \wedge \neg C)) \vee (C \wedge \neg A)$ (Associative Law)

$\equiv (A \vee A (B \wedge \neg C) \wedge (\neg B \vee (B \wedge \neg C)) \vee (C \wedge \neg A)$

$\equiv ((A \vee B) \wedge (A \vee \neg C) \wedge (\neg B \vee B) \wedge (\neg B \vee \neg C)) \vee (C \wedge \neg A)$

$\equiv ((A \vee B) \wedge (A \vee \neg C) \wedge (\neg B \vee \neg C)) \vee (C \wedge \neg A)$ [$\because \neg A \vee A \equiv T$ and $T \wedge A \equiv A$]

$\equiv ((A \vee B) \vee (C \wedge \neg A)) \wedge ((A \vee \neg C) \vee (C \wedge \neg A)) \wedge ((\neg B \vee \neg C) \vee (C \wedge \neg A))$ (Distributive Law)

$\equiv (A \vee B \vee C) \wedge (A \vee B \vee \neg A) \wedge (A \vee \neg C \vee C) \wedge (A \vee \neg C \vee \neg A) \wedge (\neg B \vee \neg C \vee C) \wedge (\neg B \vee \neg C \vee \neg A)$ (Distributive law)

$\equiv (A \vee B \vee C) \wedge (\neg A \vee \neg B \vee \neg C)$ [$\because A \vee B \vee \neg A \equiv (A \vee \neg A) \vee B \equiv T \vee B \equiv T$ and $T \wedge A = A$ etc and commutativity]

This is the required full cnf.

(ii) & (iii) → Do it yourself.

<u>Application to switching circuits</u> : An electric circuit containing only on-off switches (when a switch is on, it passes current, otherwise it does not) can be represented by a diagram in which next to each switch, we put a letter representing a necessary and sufficient condition for switches to be on.

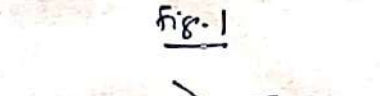


Fig. 1

Fig. 1 represents switches in series and its corresponding statement form is $A \wedge B$

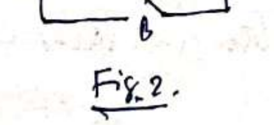


Fig. 2.

Fig. 2 represent switches in parallel and its statement form is

$A \vee B$

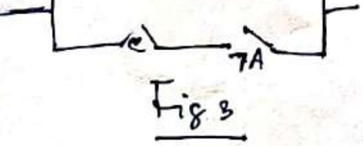


Fig 3

The condition that a current flows in the circuit in Fig 3 can be represented by the statement form

$(A \wedge B) \vee (C \wedge \neg A)$

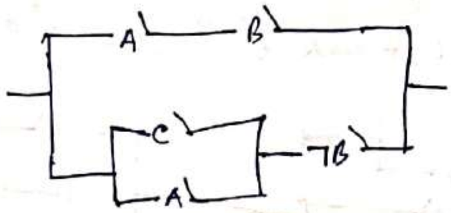


Fig. 4

The circuit represented by the circuit in Fig. 4 can be represented by a statement form $(A\wedge B)\vee((C\vee A)\wedge\neg B)$

Now, $(A\wedge B)\vee((C\vee A)\wedge\neg B)$

$\equiv ((A\wedge B)\vee(C\vee A))\wedge((A\wedge B)\vee\neg B)$ (Distributive Law)

$\equiv \cancel{((A\wedge B)(C\vee A)}\ ((A\wedge B)\vee(C\vee A))\wedge(\cancel{(A\wedge\neg B)}(A\vee\neg B)\wedge(B\vee\neg B))$ (Distributive law)

$\cancel{\equiv ((A\wedge B)(C\vee A)\wedge(A\vee\neg B)\ [\because B\vee\neg B\equiv T \text{ and } A\wedge T\equiv A]}$

$\equiv (\cancel{(A\wedge B)}\ ((A\wedge B)\vee(C\vee A))\wedge(A\vee\neg B)\ [\because B\vee\neg B\equiv T \text{ and } A\wedge T\equiv A]$

$\cancel{\equiv (((A\wedge B)\vee C)\vee A)\wedge(A\vee\neg}$

$\equiv ((A\wedge B)\vee(A\vee C))\wedge(A\vee\neg B)$ [Commutative Law]

$\equiv (((A\wedge B)\vee A)\vee C)\wedge(A\vee\neg B)$ [Associative Law]

$\equiv (A\vee C)\wedge(A\vee\neg B)$ [Absorption Law]

$\equiv A\vee(C\wedge\neg B)$ (Distributive Law)

$\equiv A\vee(\neg B\wedge C)$ (Commutative Law)

As the statement form $(A\wedge B)\vee((C\vee A)\wedge\neg B)$ is equivalent to $A\vee(\neg B\wedge C)$, the circuit in Fig. 4 is equivalent to the simple circuit shown in Fig. 5

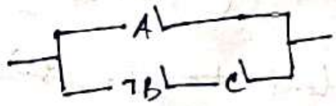


Fig. 5

Two circuits are said to be equivalent if current flows through one if and only if current flows through the other. One circuit is simpler if it contains fewer switches.

Ex.9 Find simpler equivalent circuits for those shown in the figures 6, 7 and 8 below

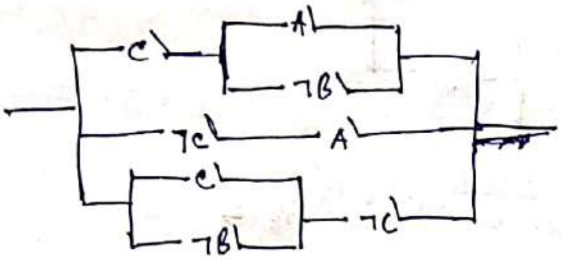


Fig. 6

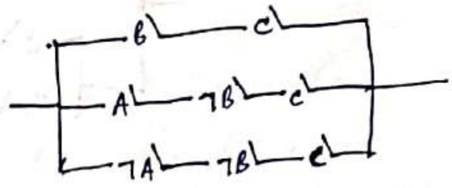
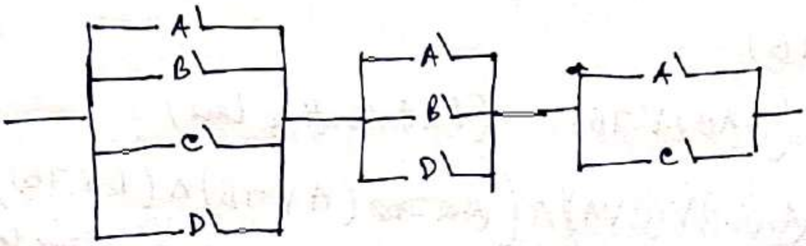


Fig. 7.



Solution In Fig. 6, the circuit can be represented by the statement form

$(C\wedge(A\vee\neg B))\vee(\neg C\wedge A)\vee((C\vee\neg B)\wedge\neg C)$

Now, $(C\wedge(A\vee\neg B))\vee(\neg C\wedge A)\vee((C\vee\neg B)\wedge\neg C)$

$\equiv (C\wedge A)\cancel{\vee(C\wedge\neg B)\vee(\neg C\wedge A)\vee(C\wedge\neg C)\vee(\neg B\wedge\neg C)}\,(\cancel{(C\vee\neg B)\wedge\neg C})$ ~~(Distributive law)~~

$\equiv (C\wedge A)\cancel{\vee(\neg C\wedge A)\vee((C\wedge\neg B)\vee(C}$

$\equiv (C\wedge A)\vee(C\wedge\neg B)\vee(\neg C\wedge A)\vee(C\wedge\neg C)\vee(\neg B\wedge\neg C)$ (Distributive law)

$\equiv (C\wedge A)\vee(\neg C\wedge A)\vee(C\wedge\neg B)\vee F\vee(\neg B\wedge\neg C)$ (Commutative law and $A\wedge\neg A\equiv F$)

$\equiv ((C\vee\neg C)\wedge A)\vee((C\wedge\neg B)\vee(\neg C\wedge\neg B))$ (Distributive law, and $A\vee F\equiv A$)

$\equiv (T\wedge A)\vee(\cancel{(C\wedge\neg C)}\,(C\vee\neg C)\wedge\neg B)$ Distributive law, and $C\vee\neg C\equiv T$

$\equiv A\vee(T\wedge\neg B)$ $(T\wedge A\equiv A)$

$\equiv A\vee\neg B$ $(T\wedge A\equiv A$

So, the circuit of Fig. 6 is equivalent to the simpler circuit given below: