6. Simplification :

$$\frac{p \wedge q}{p} \quad \text{OR, } p \wedge q \to p \text{ is a tautology}$$

$$\frac{q \wedge p}{q} \quad \text{OR, } q \wedge p \to q \text{ is a tautology}$$

7. Conjunction :

$$\frac{\begin{matrix} p \\ q \end{matrix}}{p \wedge q} \quad \text{OR, } (p \wedge q) \to (p \wedge q) \text{ is a tautology}$$

$$\frac{\begin{matrix} q \\ p \end{matrix}}{q \wedge p} \quad \text{OR, } (q \wedge p) \to (q \to p) \text{ is a tautology}$$

8. Resolution :

$$\frac{\begin{matrix} p \vee q \\ \sim p \vee r \end{matrix}}{q \vee r} \quad \text{OR, } (p \vee q) \wedge (\sim p \vee r) \to (q \to r) \text{ is a tautology}$$

9. Dilemma :

$$\frac{\begin{matrix} p \vee q \\ p \to r \\ q \to r \end{matrix}}{r} \quad \text{OR, } ((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r \text{ is a tautology}$$

10. Constructive Dilemma

$$\frac{\begin{matrix} p \vee r \\ p \to q \\ r \to s \end{matrix}}{q \vee s} \quad \text{OR, } ((p \vee r) \wedge (p \to q) \wedge (r \to s)) \to q \vee s \text{ is a tautology}$$

11. Destructive Dilemma :

$$\frac{\begin{matrix} \sim q \vee \sim s \\ p \to q \\ r \to s \end{matrix}}{\sim p \vee \sim r} \quad ((\sim q \vee \sim s) \wedge (p \to q) \wedge (r \to s)) \to \sim p \vee \sim r \text{ is a tautology}$$

Department of Mathematics, GGDC Mathematical Logic (SB)

Some problems

1. Show that the argument

$$\begin{array}{c} p \to q \\ q \to r \\ p \\ \hline r \end{array}$$

is a valid argument.

Solution: Here the premises are $p \to q$, $q \to r$ and p. The steps for deduction That r is a valid conclusion from the given premises are as follows:

1.	$p \to q$	Rule P
2.	p	Rule P
3.	q	Rule T, 1, 2 and Modus Ponens
4.	$q \to r$	Rule P
5.	r	Rule T, 3, 4 and Modus Ponens

Hence r is a valid conclusion from the given premises. It follows that the given argument is valid.

2. Show that the argument

$$\begin{array}{c} p \vee q \\ q \to r \\ p \to s \\ \sim s \\ \hline r \end{array}$$

is a valid argument.

Solution: Here the premises are $p \vee q$, $q \to r$, $p \to s$ and $\sim s$. The steps for deduction that r is a valid conclusion from the given premises are as follows,

1.	$p \to s$	Rule P
2.	$\sim s$	Rule P
3.	$\sim p$	Rule T, 1, 2 and Modus Tollens
4.	$p \vee q$	Rule P
5.	q	Rule T, 3, 4 and Disjunctive Syllogism
6.	$q \to r$	Rule P
7.	r	Rule T, 5, 6 and Modus Ponens

Hence r is a valid conclusion from the given premises. It follows that the given argument is valid.

3. Show that the argument
$$\begin{array}{c} p \to (q \to s) \\ \sim r \vee p \\ q \\ \hline r \to s \end{array}$$
is a valid argument

Solution: Here the premises are $p \to (q \to s)$, $\sim r \vee p$ and q. The steps for deduction that $r \to s$ is a valid conclusion from the given premises are as follows:

1.	$\sim r \vee p$	Rule P
2.	r	Rule P (assumed premise)
3.	p	Rule T, 1, 2 and Disjunctive Syllogism
4.	$p \to (q \to s)$	Rule P
5.	$q \to s$	Rule T, 3, 4 and Modus Ponens
6.	q	Rule P
7	s	Rule T, 5, 6 and Modus Ponens
8.	$r \to s$	Rule CP

Hence $r \to s$ is a valid conclusion from the given premises. It follows that the given argument is valid.

4. Show that s is a valid conclusion from the premises:

$p \to \sim q$, $q \vee r$, $\sim s \to p$, $\sim r$

Solution:
Here the premises are $p \to \sim q$, $q \vee r$, $\sim s \to p$ and $\sim r$. The steps for deduction that s is a valid conclusion from the given premises are as follows:

1.	$q \vee r$	Rule P
2.	$\sim r$	Rule P
3.	q	Rule T, 1, 2 and Disjunctive Syllogism

4. $p \to \sim q$ Rule P

5 $\sim(\sim q)$ Rule T and the identity $\sim(\sim q) \equiv q$

6. $\sim p$ Rule T, 4, 5 and Modus Tollens

7. $\sim s \to p$ Rule P

8. $\sim(\sim s)$ Rule T, 6, 7 and Modus Tollens

9. s Rule T and the double negation law, ($\sim(\sim s) \equiv s$)

So, s is a valid conclusion from the premises : $p \to \sim q$, $q \vee r$, $\sim s \to p$, $\sim r$

5. Show that t is a valid conclusion from the premises : $p \to q$, $q \to r$, $r \to s$, $\sim s$ and $p \vee t$

Solution: Here the premises are $p \to q$, $q \to r$, $r \to s$, $\sim s$ and $p \vee t$
The steps for deduction that t is a valid conclusion from the given premises are as follows :

1. $p \to q$ Rule P
2. $q \to r$ Rule P
3. $p \to r$ Rule T, 1, 2 and Hypothetical Syllogism
4. $r \to s$ Rule P
5. $p \to s$ Rule T, 3, 4 and Hypothetical Syllogism
6. $\sim s$ Rule P
7. $\sim p$ Rule T, 5, 6 and Modus Tollens
8. $p \vee t$ Rule P
9. t Rule T, 7, 8 and Disjunctive Syllogism

Hence t is a valid conclusion from the premises $p \to q$, $q \to r$, $r \to s$, $\sim s$ and $p \vee t$

6. Show that $r \vee s$ is a valid conclusion from the premises: $p \vee q$, $p \vee q \to \sim w$, $\sim w \to (u \wedge \sim v)$ and $(u \wedge \sim v) \to (r \vee s)$
Solution : Exercise

7. Show that $\sim p$ is tautologically implied by $\sim(p \wedge \sim q)$, $\sim q \vee r$, $\sim r$.

Solution: The steps for deduction that $\sim p$ is a valid conclusion from the given premises are as follows:

1.	$\sim(p \wedge \sim q)$	Rule P
2.	$\sim p \vee \sim(\sim q)$	Rule T, 1, De Morgan's Law
3.	$\sim p \vee q$	Rule T, 2, Double Negation Law $\sim(\sim q) \equiv q$
4.	$p \rightarrow q$	Rule T, 3, and $p \rightarrow q \equiv \sim p \vee q$
5.	$\sim q \vee r$	Rule P
6.	$q \rightarrow r$	Rule T, 5, and $p \rightarrow q \equiv \sim p \vee q$
7.	$p \rightarrow r$	Rule T, 4, 6 and Hypothetical Syllogism
8.	$\sim r$	Rule P
9.	$\sim p$	Rule T, 7, 8 and Modus Tollens

8. Show that s is a valid conclusion from the premises $p \rightarrow \sim q$, $q \vee r$, $\sim s \rightarrow p$ and $\sim r$

Solution: Here the premises are $p \rightarrow \sim q$, $q \vee r$, $\sim s \rightarrow p$ and $\sim r$. The steps for deduction that s is a valid conclusion from the given premises are as follows:

1.	$q \vee r$	Rule P
2.	$\sim r$	Rule P
3.	q	Rule T, 1, 2 and Disjunctive Syllogism
4.	$p \rightarrow \sim q$	Rule P
5.	$\sim p$	Rule T, 3, 4, and Modus Tollens
6.	$\sim s \rightarrow p$	Rule P
7.	s	Rule T, 5, 6 and Modus Tollens

Hence s is a valid conclusion from the given premises.