Notes on DSE B(1) (Linear Programming & Game Theory)

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Books followed : 1. Linear Programming and Theory of Games - P.M. Karak
2. Linear Programming and Game Theory - J.G. Chakravorty & P.R. Ghosh.
3. Linear Programming - G. Hadley
4. Linear Programming and Network flows - Mokhtar S. Bazara, John J. Jarvis and Hanif D. Sherali

Unit-1 1. Definition of Linear Programming Problem (LPP)

The main object of an industry is to produce different products in such a way that maximum profit may be earned by selling them with the limited available resources (such as raw materials, manpower, capital, power, technical appliances etc.). Similarly the main aim of the head of a family is to buy foodgrains, vegetables, fruits and other food materials for his/her family at a minimum cost which will satisfy the minimum need of the family (regarding food values, calories, proteins, vitamins etc.). The abovementioned two problems and many other similar problems can be solved by formulating them mathematically and they are called Programming problem. In a programming problem, when they are mathematically formulated, there exists a function of some finite number of variables, which is to be maximized or to be minimized, called the objectives function. Those finite number of variables are called decision variables. Also the maximum or minimum value is found under a finite number of constraints which are ~~f~~ also functional equations or inequations of the decision variables. If in a Programming problem, the objective function is linear and all the constraints are ~~linear~~ linear equations or inequations then it is called a linear programming problem (LPP). At the time of World war II, a group of Mathematicians was trying to solve the LPP. as the government was trying to utilize their resources most efficiently. The team led by George B. Dantzig found a method, called simplex method to solve LPP.

2. Formation of LPP from daily life involving inequations

Let us formulate some linear programming problem (LPP) from daily life as follows:

2.1 Production Allocation problem:

Example 1 Four different metals namely, iron, copper, zinc and manganese are required to produce three commodities A, B and C. 40 kg. iron, 30 kg. copper, 7 kg. zinc and 4 kg. manganese are needed to produce one unit of A. 70 kg. iron, 14 kg. copper and 9 kg. magnanese are needed to produce one unit of B. Similarly, to produce one unit of C, 50 kg. iron, 18 kg copper and 8 kg. zinc are required. The total available quantities of metals are 1000 kg iron, 500 kg. of copper and 200 kgs of zinc and manganese each. The profit in selling a unit of A, B and C respectively are Rs. 300, Rs. 200 and Rs. 100. Formulate the problem mathematically.

Solution: Let z be the total profit and the problem here is to maximize z.

All the available quantities of metals and the quantities required to produce A, B and C are given below in a tabular form.

	Iron	Copper	Zinc	Manganese
Total	1000 kg	500 kg	200 kg	200 kg
A	40 kg	30 kg	7 kg	4 kg
B	70 kg	14 kg	0 kg	9 kg
C	50 kg	18 kg	8 kg	0 kg

Let x_1 units of A, x_2 units of B and x_3 units of C be be produced. So, the objective function is

the profit $z = 300x_1 + 200x_2 + 100x_3$ in Rs.

Here the total quantity of iron needed is $40x_1 + 70x_2 + 50x_3$ in kg.

Total quantity of copper needed is $30x_1 + 14x_2 + 18x_3$ in kg.

Total quantity of Zinc needed is $7x_1 + 0 \cdot x_2 + 8x_3 = 7x_1 + 8x_3$ in kg.

Total quantity of Manganese needed is $4x_1 + 9x_2 + 0 \cdot x_3 = 4x_1 + 9x_2$ in kg

By the given conditions of the problem,

$$
\begin{aligned}
40x_1 + 70x_2 + 50x_3 &\leq 1000 \\
30x_1 + 14x_2 + 18x_3 &\leq 500 \\
7x_1 \qquad + 8x_3 &\leq 200 \\
4x_1 + 9x_2 \qquad &\leq 200
\end{aligned}
$$

So, we have to find x_1, x_2, x_3 which would maximize $z = 300x_1 + 200x_2 + 100x_3$

We write this as Maximize $z = 300x_1 + 200x_2 + 100x_3$

So, the problem can be written as

Maximize $z = 300x_1 + 200x_2 + 100x_3$

subject to