$$z_1 - c_1 = \frac{7\times 6 - (-4)\times 4}{7} = \frac{58}{7}$$

$$z_2 - c_2 = \frac{7\times 0 - (-4)\times 1}{7} = \frac{4}{7}$$

$$z_3 - c_3 = 0$$

$$z_4 - c_4 = 0$$

So, all $z_j - c_j \geq 0$, $j = 1,2,3,4$. So, optimality condition is satisfied.

Here an optimal solution is

$$x_1 = 0,\ x_2 = 0,\ x_3 = 23/7;\ x_4 = \frac{4}{7}$$

and the optimal value of the objective function $= \dfrac{7\times 6 - (-4)\times 4}{7} = \dfrac{58}{7}.$

$$= 0 - 0 + 2\cdot\frac{23}{7} + \frac{3\cdot 4}{7} = \frac{58}{7}$$

4.6. Simplex Algorithm

Here we do the calculations of the simplex method in tabular form and here also we find an initial basic feasible solution in a different way:

Case I <u>When all constraints of the LPP are '≤' type all $b_i \geq 0$,</u>: We first convert them in equations by adding slack variables one to each inequation. If there are m constraints inequations with initial n variables, the converted equations contain $(m+n)$ variables of which m variables are slack variables.

The converted equations are:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + x_{n+1} = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + x_{n+2} = b_2$$

$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + x_{n+m} = b_m$$

So the coefficient matrix A of the above equations is

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & \overbrace{1 \;\; 0 \cdots 0}^{m \text{ slack vectors}} \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 \;\; 1 \cdots 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & 0 \;\; 0 \cdots 1 \end{bmatrix}$$

It is an $m \times (m+n)$ matrix

The column vectors associated with slack variables are known as slack vectors. All slack vectors are unit vectors which are linearly independent. Hence the m slack vectors constitute a basis matrix

$$B = (e_1, e_2, \cdots e_m) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{m \times m} = I_m,$$ the identity

matrix of order m. The corresponding basic variables all slack variables, say, x_B. As $B = I_m$,

$B^{-1} = I_m$. So, $x_B = B^{-1}b = I_m^{-1}b = I_m b = b \geq 0$

as $b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$ and all $b_i \geq 0$

So, $x_B = b$ becomes the initial basic feasible solution for this case where

$$x_B = [x_{n+1}, x_{n+2}, \ldots, x_{n+m}]$$

Case 2 When the constraints are of mixed type connected with '$\leq$' and '$\geq$' and '$=$', and all b_i may not be non-negative

Initially we make all $b_i \geq 0$ by suitable adjustment of sign and for this '$\leq$' type changes to '$\geq$' type and vice versa and no change for equality. Then we convert all inequations into equations by introducing slack and surplus variables. ~~By those cons~~ The column vectors associated with the surplus variable is called surplus vector. In general, in this type of problem, we do not get initial basis matrix an ~~unit~~ a identity matrix with help of the slack columns. We forcefully introduce some artificial variables in the constraints, such that alongwith the corresponding slack columns (if any), the columns ~~of~~ the artificial variables forms ~~an~~ a basis which is an identity matrix. The problem is changed but we will show later that in this case we can ~~conc~~ conclude about the solution of the original problem.

Construction of the initial simplex table

In both the cases mentioned earlier, we get an unit basis matrix $B = I_m$ which is present in the coefficient matrix will all $b_i \geq 0$. Then, an initial BFS is $x_B = B^{-1}b = I_m^{-1}b = b \geq 0$ and $z = c_B x_B = c_B b$ (initial value of the objective function.) and $y_j = B^{-1}a_j = I_m^{-1} a_j = a_j$

Then $y_{ij} = a_{ij}$ for all values of i and j. Hence the elements can be determined easily from the coefficient matrix.

Now $z_j - c_j = c_B B^{-1} a_j - c_j = c_B y_j - c_j = c_B a_j - c_j$

With this data an initial simplex table can be constructed as shown in the table 1 below

		C	c_1	c_2	c_3	...	Min ratio
B	C_B	b	a_1	a_2	a_3	...	
β_1	C_{B_1}	b_1	a_{11}	a_{12}	a_{13}	...	
β_2	C_{B_2}	b_2	a_{21}	a_{22}	a_{13}	...	
$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	
β_m	C_{B_m}	b_m	a_{m1}	a_{m2}	a_{m3}	...	
$z_j - c_j$		z_0	$z_1 - c_1$	$z_2 - c_2$	$z_3 - c_3$	...	

Note: $\beta_1, \beta_2, \ldots \beta_m$ are the m unit column vectors in order in the identity Basis matrix. z_0 is the initial value of the value of the objective function

4.7 Computational procedure (when m slack vectors constitute the initial Basis $B = I_m$) and the problem is of maximization (as every problem can be converted to maximization problem) and $b \geq 0$

1. Convert all constraints into equation by adding m slack vectors, one to each constraint.
2. Readjust the objective function accordingly.
3. Form the initial simplex table as described earlier.