3. If all $z_j - c_j \geq 0$, the initial BFS is optimal and thus the value of the objective function will be the last element of the column headed by b. If at least one $z_j - c_j < 0$, the solution is not optimal, then proceed further to construct the next table to get the next BFS which may ~~offt~~ give optimal value of the objective function. For this, we have to select the key elem y_{rk}. [If $z_k - c_k$ is minimum (negative), the vector a_k will enter the new basis replacing the vector in the rth position under the column vector in the basis and c_k will replace c_{B_r}. If the minimum is not unique select any one of them. The value of r is to be determined by Min. ratio rule which has been discussed in Page-54.

4. Construct the next table as follows: Divide the elements y_{rj} [$j = 0, 1, 2, \ldots, n$] of the rth row by of the previous table y_{rk} to get the rth row of the next table.

$$\text{Then } y'_{rj} = \frac{y_{rj}}{y_{rk}} \quad i = r,\ j = 0, 1, 2, \ldots, n$$

and to get the elements of the other rows of the table, we have

$$y'_{ij} = y_{ij} - y_{ik}\frac{y_{rj}}{y_{rk}}$$

$$= \frac{1}{y_{rk}} \begin{vmatrix} y_{ij} & y_{rj} \\ y_{ik} & y_{rk} \end{vmatrix} \quad i=1,2,\dots m,\ i \neq r \text{ and } i=m+1 \text{ (for } z_j - c_j)$$

The value of z and $z_j - c_j$, $j=1,2,\dots,n$ are given the last ($(m+1)$th) row. If $z_j - c_j \geq 0$ for all j, the BFS is optimal. If at least one $z_j - c_j < 0$, then proceed further to ~~construct~~ construct a new table to get an optimal solution an so on until the conditions $z_j - c_j \geq 0$ are satisfied for all j. At the optimal table, the optimal value of the objective function is the last element under the column headed by b or we can use $z = C_B x_B$ at the optimal stage.

5. If at any stage $z_j - c_j < 0$ for at least one column with $y_{ij} \leq 0$ for all i, the problem has no finite solution, i.e the problem is said to have an unbounded solution.

6. If optimality is satisfied, i.e., $z_j - c_j \geq 0$ for all j and $z_j - c_j = 0$ corresponding to some non-basis vector then multiple optimal solutions exist. Another new optimal solution is found out by choosing a new basis taking this non basis

vector in the basis.

Note: We show that if the LPP Maximize $z = cx$
Subject to $Ax = b$
$x \geq 0$

has two optimal then it has infinite number of optimal solutions.

Proof: Let x_1 and x_2 be two optimal solutions of the LPP. Then $Ax_1 = b$ and $x_1 \geq 0$
, $Ax_2 = b$ and $x_2 \geq 0$

and $z_0 = cx_1 = cx_2 \leq cx$ for any x such that $Ax = b$ and $x \geq 0$

Let $y = \lambda x_1 + (1-\lambda) x_2$, $0 < \lambda < 1$

Then $y \geq 0$ as $x_1 \geq 0$, $x_2 \geq 0$, $\lambda > 0$ and $(1-\lambda) > 0$

Also $Ay = A(\lambda x_1 + (1-\lambda) x_2) = \lambda(Ax_1) + (1-\lambda) A x_2$
$= \lambda b + (1-\lambda) b = b$

Also $cy = c(\lambda x_1 + (1-\lambda) x_2)$
$= \lambda (cx_1) + (1-\lambda) cx_2$
$= \lambda z_0 + (1-\lambda) z_0$
$= z_0$

So, $y = \lambda x_1 + (1-\lambda) x_2$, $0 < \lambda < 1$ is also ~~and~~ an optimal solution. As $0 < \lambda < 1$.
So, the ~~[illegible]~~ number of optimal solution is infinite.

Example 1 Solve the LPP

$$\text{maximize } z = 5x_1 + 2x_2 + 2x_3$$

subject to
$$x_1 + 2x_2 - 2x_3 \leq 30$$
$$x_1 + 3x_2 + x_3 \leq 36, \quad x_1, x_2, x_3 \geq 0$$

Solution: This is a maximization problem. Here $b_i \geq 0$ for $i=1,2$ and the constraints involved with sign '$\leq$'.

Introducing two slack variables x_4 and x_5 one to each constraint, we get the following converted equations

$$x_1 + 2x_2 - 2x_3 + x_4 = 30$$
$$x_1 + 3x_2 + x_3 + x_5 = 36$$
$$x_i \geq 0, \quad i=1,2,3,4,5$$

The adjusted objective function is

$$Z = 5x_1 + 2x_2 + 2x_3 + 0.x_4 + 0.x_5$$

So, the initial simplex table (Table 1) is

Table 1

		C	5	2	2	0	0	Min. ratio
B	C_B	b	a_1	a_2	a_3	a_4	a_5	
a_4	0	30	$\boxed{1}$	2	−2	1	0	$30/1 = 30$*
a_5	0	36	1	3	1	0	1	$36/1 = 36$
	$z_j - c_j$	0	-5^*	−2	−2	0	0	

Here optimality is not satisfied as z_1-c_1, z_2-c_2 and z_3-c_3 are all negative and at least one component of y_1, y_2, y_3 are positive and

$$\min\{z_1-c_1, z_2-c_2, z_3-c_3\} = -5 \text{ which is } z_1-c_1$$

So, a_1 is the entering vector in the new basis

Now $\min\limits_{i=1,2}\left\{\dfrac{x_{Bi}}{y_{i1}}, y_{i1}>0\right\} = \min\left\{\dfrac{30}{1}, \dfrac{36}{1}\right\} = 30$

which occurs for $i=1$. So, the departing vector is a_4. So, the intersection of 1st row and 1st column, i.e $y_{11} = 1$ is the key element

So, we construct the next table as