$$2x_1 + x_2 + 3x_3 \geq 40$$

and $x_1 + 3x_2 + 2x_3 \geq 35$. Here also $x_i \geq 0$, $i = 1, 2, 3$

Hence the problem can be written as,

~~Maximize~~ Minimize $Z = 12x_1 + 10x_2 + 13x_3$

Subject to

$$x_1 + 2x_2 + x_3 \geq 25$$
$$2x_1 + x_2 + 3x_3 \geq 40$$
$$x_1 + 3x_2 + 2x_3 \geq 35$$
$$x_i \geq 0, \ i = 1, 2, 3$$

2.3 Transportation problem

Example 3 A company has factories at O_1, O_2 and O_3 which supply fertilizer to warehouses at D_1, D_2, D_3 and D_4. Monthly factory productions of fertilizer ~~of~~ at O_1, O_2 and O_3 respectively are 160, 150 and 190 in tons. Monthly requirement of the warehouses at D_1, D_2, D_3 and D_4 respectively are 80, 120, 110 and 190 in tons. The cost of transportation per ton from factory O_i to warehouse D_j are given in a matrix form as follows (in Rupees) $\{i = 1, 2, 3,\ j = 1, 2, 3, 4\}$:

	D_1	D_2	D_3	D_4
O_1	420	480	380	370
O_2	400	490	520	510
O_3	390	380	400	430

The problem is to supply the fertilizer from the factories to the warehouses so that the cost of transportation is minimumum. Formulate this problem mathematically

Solution: Let x_{ij} be the amount of fertilizer in tons to be ~~transf~~ transported from factory O_i to warehouse D_j,

$i=1,2,3$ and $j=1,2,3,4$. If Z be the objective function, then Z is the total cost of transportation. So,

$$Z = 420x_{11} + 480x_{12} + 380x_{13} + 370x_{14} + 400x_{21} + 490x_{22} + 520x_{23} + 510x_{24} + 390x_{31} + 380x_{32} + 400x_{33} + 430x_{34}$$ in Rupees

By the given conditions of the problem,

$$x_{11} + x_{12} + x_{13} + x_{14} = 160$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 150$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 190$$
$$x_{11} + x_{21} + x_{31} = 80$$
$$x_{12} + x_{22} + x_{32} = 120$$
$$x_{13} + x_{23} + x_{33} = 110$$
$$x_{14} + x_{24} + x_{34} = 190$$

and $x_{ij} \geq 0$, $i=1,2,3$ and $j=1,2,3,4$

So, the problem can be written as

Minimize $Z = 420x_{11} + 480x_{12} + 380x_{13} + 370x_{14} + 400x_{21} + 490x_{22} + 520x_{23} + 510x_{24} + 390x_{31} + 380x_{32} + 400x_{33} + 430x_{34}$

Subject to,

$$x_{11} + x_{12} + x_{13} + x_{14} = 160$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 150$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 190$$
$$x_{11} + x_{21} + x_{31} = 80$$
$$x_{12} + x_{22} + x_{32} = 120$$
$$x_{13} + x_{23} + x_{33} = 110$$
$$x_{14} + x_{24} + x_{34} = 190$$

and $x_{ij} \geq 0$, $i=1,2,3$ and $j=1,2,3,4$

Miscellaneous Problems (2.4)

Example 4 An electronic company manufactures two radio models each on a separate production line. The daily capacity of the first line is 60 radios and that of the second is 75 radios. Each unit of the first model uses 10 pieces of electronic component A and 15 pieces of electronic component B., whereas each unit of the second model requires 8 pieces of electronic component A and 16 pieces of electronic component B. The daily availability of the electronic component A is 800 and that of electronic component B is 1000. The profit per unit of models 1 and 2 are Rs. 500 and Rs 400 respectively. Formulate this problem as an LPP so as to maximize the profit.

Solution: Let x_1 and x_2 be the number of two radio models produced daily each on a separate production line. If Z be the objective function, Then

$$Z = 500x_1 + 400x_2 \quad \text{in Rs.}$$

Since the daily capacity of the first line and second line are 60 and 75 respectively, then we have

$$x_1 \leq 60$$
$$x_2 \leq 75$$

The total ~~cost~~ electronic component A required $= 10x_1 + 8x_2$

The total electronic component B required $= 15x_1 + 16x_2$

From the conditions of the problem given, we have

$$10x_1 + 8x_2 \le 800$$

and $\quad 15x_1 + 16x_2 \le 1000$

Hence the problem can be written as

Maximize $z = 500x_1 + 400x_2$

Subject to

$$x_1 \le 60$$
$$x_2 \le 75$$
$$10x_1 + 8x_2 \le 800$$
$$15x_1 + 16x_2 \le 1000$$

and $x_1 \geq 0,\ x_2 \geq 0$

Example 5 A hospital has the following minimum requirement for nurses:

Period	Clock time (24 hours a day)	Minimum number of nurses required
1	6 AM – 10 AM	60
2	10 AM – 2 PM	70
3	2 PM – 6 PM	60
4	6 PM – 10 PM	50
5	10 PM – 2 AM	20
6	2 AM – 6 AM	30

Nurses report to the hospital wards at the beginning of each period and work for eight consecutive hours. The hospitals wants to determine the minimum number of nurses so that there may be sufficient number of nurses available for each period. Formulate this as an LPP.

Solution : Let $x_1, x_2, \ldots, x_6$ be the number of nurses required for the period $1, 2, \ldots, 6$. Then the objective function is

$$z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$. It is a minimization problem.

Now the constraints can be written as follows:
As there is x_1 nurses for the period 1 and 2 and x_2 nurses work for the period 2 and 3 etc. . So, for period 2, $x_1 + x_2 \geq 70$

Similarly,

$$x_2 + x_3 \geq 60$$
$$x_3 + x_4 \geq 50$$
$$x_4 + x_5 \geq 20$$
$$x_5 + x_6 \geq 30$$
$$x_6 + x_1 \geq 60$$

and $x_j \geq 0,\ j = 1, 2, \ldots, 6$