Artificial variables technique is used in that case when we are not getting any unit basis as initial basis matrix, when $b \geq 0$. But there is a problem. We are introducing an artificial variable (≥ 0) in an equation. So, the equation will be satisfied only if the values of the artificial variables be equal to zero. So in solving the problem of this type by using artificial variables ~~be equal to zero~~ by using simplex method, one must be sure at the optimal stage, that all artificial variables are at zero level. If it is not possible to bring all artificial variables at zero level at the optimal stage, we conclude that the problem has no feasible solution.

In an attempt to solve the problem, by using simplex method, the following three cases may arise when the conditions are satisfied (at the optimal stage):

1. All artificial vectors are not present in the basis which indicates that all artificial variables are at zero level at the optimal stage. Thus the solution obtained is a BFS.

2. Some artificial vectors are present in the basis and some artificial variables are at positive level at the optimal stage. In that case there exists no feasible solution to the problem.

3. All artificial variables are at zero but least one artificial vector present in the basis at the optimal

stage. Here the solution under test is an optimal solution. Here the ~~constraints~~ converted equations are consistent but some of the constraints may be redundant. By redundancy, we mean that the system of equation are linearly dependent.

There are two methods of solving problems of this type : (a) Charne's Big-M method or method of penalties

(b) Two phase method.

4.8 Charnes Big M method

In this method, initially, minimum number of artificial variables are to be inserted in the equations to get a unit basis matrix from the coefficient matrix. To each of the artificial variables, a very high negative price or cost, say $-M$ (M is positive and very large such that a determines the the sign of the expression $aM+b$, a, b are real numbers) is attached as cost coefficient in the objective function of the problem. Due to very high negative ~~prices~~ costs, the objective function can not be improved in the presence of artificial variables. With the negative high costs, we proceed to solve the problem in usual method and following cases may arise :

1. At any stage, all artificial vectors may be driven out from the basis. In that case, all artificial variables are at zero level at that stage. Now if the optimality conditions are satisfied at that stage, the

~~the~~ problem is solved and the problem has an optimal solution. If the optimality conditions are not satisfied we may proceed further to get an optimal solution ~~[illegible]~~ omitting all column vectors corresponding to the artificial variables. The artificial vectors are used only as agents to get a unit basis.

2. Some artificial variables are at positive level, though the optimality conditions are satisfied. In that case the problem has no feasible solution at all. Here some artificial ~~[illegible]~~ vectors must be present at the final stage.

3. All artificial variables are at zero level and at least one artificial vector is present in the basis. If at this stage, the optimality conditions are satisfied, the solution obtained is an optimal solution. Here one or more constraints may be redundant. If the optimality conditions are not satisfied, proceed further as in the previous cases to get an optimal solution.

Note: In this connection it is important to note that once an artificial vector leaves the basis, we forget ~~at~~ all about it forever and never consider it as a vector to enter into the basis at any next iteration. The rules of construction of the tables are same as given in the previous cases.

Example 1 Solve the following LPP:

$$\text{Minimize } Z = 4x_1 + 3x_2$$

subject to

$$x_1 + 2x_2 \geq 8$$

$$3x_1 + 2x_2 \geq 12, \quad x_1, x_2 \geq 0$$

by Charnes Big M method.

Solution: The problem here is to minimize z. Here $b \geq 0$ where $b = \begin{pmatrix} 8 \\ 12 \end{pmatrix}$. Let $z' = -z$ then

$\min z = -\max(-z) = -\max z'$

Hence the problem is to maximize $z' = -4x_1 - 3x_2$

Introducing two surplus variables x_3 and x_4 we get the following converted equations

$$x_1 + 2x_2 - x_3 = 8$$
$$3x_1 + 2x_2 - x_4 = 12$$

The coefficient matrix does not contain a unit basis matrix.

To get a unit basis matrix, two artificial variables x_5 and x_6 are added, one to each equation and the equations are

$$x_1 + 2x_2 - x_3 + x_5 = 8$$
$$3x_1 + 2x_2 - x_4 + x_6 = 12$$

Now the coefficient matrix contains a unit basis.

The adjusted objective function z' is given by

$$z' = -4x_1 - 3x_2 + 0.x_3 + 0.x_4 - Mx_5 - Mx_6$$

[assigning a very large negative cost to each of the artificial variables x_5 and x_6, M is a large positive number]

So, the simplex tables are: