minimized. Then put $U' = -x_6 - x_7$ and $\min U = -\max U'$

Hence the auxiliary LPP is to maximize U' where

$$U' = 0.x_1 + 0.x_2 + 0.x_3 + 0.x_4 + 0.x_5 - x_6 - x_7$$

Simplex tables of the auxiliary LPP

		c	0	0	0	0	0	-1	-1	
B	c_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	Min ratio
a_6	-1	8	1	$\boxed{2}$	-1	0	0	1	0	$8/2 = 4^*$
a_7	-1	12	3	2	0	-1	0	0	1	$12/2 = 6$
a_5	0	60	5	6	0	0	1	0	0	$60/6 = 10$
$z_j - c_j'$		-18	-4	-4^*	1	1	0	0	0	
a_2	0	4	1/2	1	-1/2	0	0	1/2	0	$4/\frac{1}{2} = 8$
a_7	-1	4	$\boxed{2}$	0	1	-1	0	-1	1	$4/2 = 2^*$
a_5	0	36	2	0	3	0	1	-3	0	$36/2 = 18$
$z_j - c_j'$		-4	-2^*	0	-1	1	0	2	0	
a_2	0	3	0	1	-3/4	1/4	0	3/4	-1/2	
a_1	0	2	1	0	1/2	-1/2	0	-1/2	1/2	
a_5	0	32	0	0	2	1	1	-2	-1	
$z_j - c_j'$		0	0	0	0	0	0	1	1	

Hence $z_j - c_j' \geq 0$ for all j. Hence the optimality conditions are satisfied and $\min U = -\max U' = -0 = 0$

Here all artificial ~~variables are non-basic~~ vectors are driven out from the basis and $\min U = 0$. Thus we proceed to solve the original LPP after removing the columns corresponding to the artificial variables

2nd phase

The objective function of the original LPP is $z = 3x_1 + 5x_2$ and the problem is to minimize it. Then put $z' = -3x_1 - 5x_2$ and $\min z = -\max z'$. Hence the problem is ultimately a problem to maximize

$$z' = -3x_1 - 5x_2 + 0.x_3 + 0.x_4 + 0.x_5$$

Initial simplex table of the original LPP

		c	-3	-5	0	0	0
B	C_B	b	a_1	a_2	a_3	a_4	a_5
a_2	-5	3	0	1	$-3/4$	$1/4$	0
a_1	-3	2	1	0	$1/2$	$-1/2$	0
a_5	0	32	0	0	2	1	1
$z_j - c_j'$		-21	0	0	$9/4$	$1/4$	0

As $z_j - c_j \geq 0$ for all j, the optimality conditions are satisfied.

Hence $\max z' = -21$ at $x_1 = 2$ and $x_2 = 3$

or, $\min z = -\max z' = 21$ at $x_1 = 2,\ x_2 = 3$.

Example 2 Solve the following LPP by two phase method:

Maximize $z = 5x_1 + 11x_2$

Subject to $2x_1 + x_2 \leq 4$

$3x_1 + 4x_2 \geq 24$

$2x_1 - 3x_2 \geq 6$, $x_1, x_2 \geq 0$

Solution: After introduction of slack variable, surplus variables and artificial variables, the constraints are

$$
\begin{aligned}
2x_1 + x_2 + x_3 \qquad\qquad\qquad\qquad &= 4 \\
3x_1 + 4x_2 \quad - x_4 \qquad + x_6 \qquad &= 24 \\
2x_1 - 3x_2 \qquad\quad - x_5 \qquad + x_7 &= 6,
\end{aligned}
$$

$x_1, x_2 \geq 0$, $x_3 \geq 0$ is a slack variable, $x_4, x_5 \geq 0$ are surplus variables and $x_6, x_7 \geq 0$ are artificial variables. ~~In the~~

In the first phase, the objective function of the auxiliary LPP is $U = x_6 + x_7$ and we are to minimize it. Put $U' = -x_6 - x_7$ and $\min U = -\max U'$. So, we have to

maximize $U' = 0.x_1 + 0.x_2 + 0.x_3 + 0.x_4 + 0.x_5 - x_6 - x_7$

Phase I

Simplex tables of the auxiliary ~~function~~ LPP

		c	0	0	0	0	0	−1	−1	
B	C_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	Min ratio
a_3	0	4	$\boxed{2}$	1	1	0	0	0	0	$4/2 = 2^*$
a_6	−1	24	3	4	0	−1	0	1	0	$\frac{24}{3} = 8$
a_7	−1	6	2	−3	0	0	−1	0	1	$6/2 = 3$
$z_j - c_j$		−30	-5^*	−1	0	1	1	0	0	
a_1	0	2	1	$1/2$	$1/2$	0	0	0	0	
a_6	−1	18	0	$5/2$	$-3/2$	−1	0	1	0	
a_7	−1	2	0	−4	−1	0	−1	0	1	
$z_j - c_j$		−20	0	$3/2$	$5/2$	1	1	0	0	

$a_1 \to$ entering vector
$a_3 \to$ departing vector
$2 \to$ key element

As $z_j - c_j \geq 0$ for all j in the second table,
and $\max U' = -20$, i.e., $\min U = -(-20) = 20$ and
one or more artificial variables are at positive level ~~in the basic solution~~ as basic components
So, the only conclusion is that the original problem has no feasible solution and we need not go further to phase II.

Example 3 Solve the following LPP by two phase method:

$$\text{Minimize } z = 4x_1 + x_2$$

subject to
$$x_1 + 2x_2 \leq 3$$
$$4x_1 + 3x_2 \geq 6$$
$$3x_1 + x_2 = 3 \quad \text{and } x_1, x_2 \geq 0$$

Solution: Introducing slack variable x_3, surplus variable x_4 and artificial variables x_5 and x_6, the constraints are

$$x_1 + 2x_2 + x_3 = 3$$
$$4x_1 + 3x_2 - x_4 + x_5 = 6$$
$$3x_1 + x_2 + x_6 = 3$$
$$x_i \geq 0, \; i = 1,2,3,4,5,6$$

Phase I

As there are two artificial variables, x_5 and x_6, our problem is to find the minimum value of the auxiliary objective function

$$U = x_5 + x_6 = 0.x_1 + 0.x_2 + 0.x_3 + 0.x_4 + x_5 + x_6$$

subject to the constraints given above. Now let

$$U' = -U \quad \text{and} \quad \min U = -\max(-U) = -\max U'$$

Phase I Simplex tables

B	C_B	c → b	0 → a_1	0 → a_2	0 → a_3	0 → a_4	−1 → a_5	−1 → a_6	Min. Ratio
a_3	0	3	1	2	1	0	0	0	$3/1 = 3$
a_5	−1	6	4	3	0	−1	1	0	$6/4 = 2$
a_6	−1	3	$\boxed{3}$	1	0	0	0	1	$3/3 = 1^*$
$z_j - c_j$		−9	-7^*	−4	0	1	0	0	
a_3	0	2	0	$\boxed{5/3}$	1	0	0	$-1/3$	$\frac{2}{5/3} = 6/5^*$
a_5	−1	2	0	$5/3$	0	−1	1	$-4/3$	$\frac{2}{5/3} = 6/5$
a_1	0	1	1	$1/3$	0	0	0	$1/3$	$\frac{1}{1/3} = 3$
		−2	0	$-5/3^*$	0	1	0	$7/3$	
a_2	0	$6/5$	0	1	$3/5$	0	0	$-1/5$	
a_5	−1	0	0	0	−1	−1	1	−1	
a_1	0	$3/5$	1	0	$-1/5$	0	0	$2/5$	
		0	0	0	1	1	0	2	

All $z_j - c_j \geq 0$, so optimality is reached.

So, $\text{Min } U = -\text{Max } U' = 0$. But here one artificial variable x_5 is at zero level. So, there may be redundancy