in the constraints. We shall start the phase II assuming the cost of the artificial variable x_5 be zero.

and Phase II simplex tables

Now we shall have to find the minimum value of z.

Now $\min z = -\max(-z) = -\max\left(-4x_1 - x_2 + 0.x_3 + 0.x_4 + 0.x_5\right)$

		c	-4	-1	0	0	0
B	C_B	b	a_1	a_2	a_3	a_4	a_5
a_2	-1	6/5	0	1	3/5	0	0
a_5	0	0	0	0	-1	-1	1
a_1	-4	3/5	1	0	-1/5	0	0
$z_j - c_j'$		$-\frac{18}{5}$	0	0	1/5	0	0

$z_j - c_j' \geq 0$ for all j. Hence optimality condition is being satisfied.

But $z_4 - c_4 = 0$ corresponding to a non-basic variable.

The alternative optimal solution exists.

$\text{Max}(-z) = -\frac{18}{5}$. So, $\min z = -\text{Max}(-z) = \frac{18}{5}$

at $x_1 = 3/5$, $x_2 = 6/5$

We will discuss the problem of degeneracy in LPP and its resolution as last section of the Notes.

5. Duality Theory

To every LPP there corresponds another LPP. If the first is called primal problem then the other is called dual problem and vice versa. It will be seen later that every primal has a unique dual problem. If the primal is of maximization, then the dual is of minimization and vice versa. That is, if someone wants to maximize his/her profit, correspondingly there is someone who ~~wants~~ wants to minimize his/her cost and similarly if someone wants to minimize his/her cost there is someone who wants to maximize his/her profit.

Also we will see that while solving the primal LPP, we are at the same time solving the dual LPP also.

To make the concept more clear, let us consider following problem:

Type of food	Units of vitamin present per gram		Cost prices per gram of the foods
	A	B	
X	5	6	Rs. 1
Y	8	10	Rs. 2
Minimum requirement of vitamin in gram	80	100	

Let x_1 grams of the food X and x_2 grams of the food Y be purchased and the problem is to minimize the cost.

Now the LPP becomes

$$\text{Minimize } Z = x_1 + 2x_2$$

$$\text{subject to } \quad 5x_1 + 8x_2 \geq 80$$

$$6x_1 + 10x_2 \geq 100$$

$$\text{and } x_1, x_2 \geq 0$$

If we consider it as a primal problem, consider the corresponding problem as follows: Let a dealer sells vitamin A and vitamin B required for two types of foods X and Y. His problem is ~~of~~ to fix the cost per unit of vitamins A and B in such a way that the prices per gram of X and Y do not exceed the amount mentioned above. His problem is also to get a maximum profit in selling the vitamins

Let v_1 and v_2 be the prices of per unit of vitamin A and vitamin B respectively. Therefore his problem is to

$$\text{Maximize } W = 80v_1 + 100v_2$$

Subject to $5v_1 + 6v_2 \leq 1$

$8v_1 + 10v_2 \leq 2$

and $v_1, v_2 \geq 0$

This problem is called the dual of the first problem. Similarly if you consider the second problem as primal problem then the first problem is the dual problem.

Here we consider the maximization problem as the primal problem

Standard Symmetric form of primal problem

The following LPP

Maximize $Z = cx$
Subject to $Ax \leq b$
$x \geq 0$

where $c = (c_1, c_2, \ldots c_n)$ a row vector and

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad A = [a_{ij}]_{m \times n} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}.$$

is called the standard symmetric primal problem.

Then we define the dual as

Minimize $W = b^t w$
subject to $A^t w \geq c^t$
$w \geq 0$

where b^t, A^t, c^t are the transpose of b, A and c

and $w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix}$

<u>Theorem 6.1</u> The dual of the dual is primal

Proof: Let the primal problem be

$$\begin{aligned} &\text{Max } z = cx \\ &\text{subject to } Ax \leq b \\ &x \geq 0 \end{aligned} \quad \cdots (1)$$

Then its dual is

$$\begin{aligned} &\text{Min } W = b^t w \\ &\text{subject to } A^t w \geq c^t \\ &w \geq 0 \end{aligned} \quad \cdots (2)$$