So, the problem can be written as the following LPP:

$$\text{Minimize } Z = x_1+x_2+x_3+x_4+x_5+x_6$$

Subject to
$$\begin{aligned} x_1+x_2 &\geq 70 \\ x_2+x_3 &\geq 60 \\ x_3+x_4 &\geq 50 \\ x_4+x_5 &\geq 20 \\ x_5+x_6 &\geq 30 \\ x_6+x_1 &\geq 60 \end{aligned}$$

and $x_j \geq 0$, $j=1,2,\dots,6$.

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Exercises (Formulate the following problems mathematically)

Exercise 1 A manufacturer of leather belts makes three types of Belts, Belt A, Belt B and Belt C. These belts are processed on three machines $M_1$, $M_2$, and $M_3$. Belt A requires 2 hours on Machine $M_1$ and 3 hours on Machine $M_3$. Belt B requires 3 hours on Machine $M_1$, 2 hours of Machine $M_2$ and 2 hours on Machine $M_3$ and Belt C requires 5 hours on Machine $M_2$ and 4 hours on Machine $M_3$. There are 12 hours of time per day available on Machine $M_1$, 16 hours of time per day available on Machine $M_2$ and 20 hours of time ~~available on Mach~~ per day available on machine $M_3$. The profit per unit of Belt A, Belt B and Belt C are Rs 25, Rs 35 and Rs 40 respectively. Formulate this problem mathematically for the daily production so as to maximize the profit.

Exercise 2 The daily minimum requirement of a patient for vitamins A and B are 100 units and 120 units respectively.

There are two foods, X and Y ~~are~~ available in the market which Contains vitamins A and B. Food X contains 6 units of vitamin A and 7 units of vitamin B per gram. Food Y contains 8 units of vitamin A and 12 units of vitamin B per gram. ~~Formulate the problem as an LPP to minimize~~ the ~~Food X is~~ The cost of Food X is Rs 2 per gram and Food Y is Rs. 3 per gram. Formulate the problem as an LPP to minimize the cost.

Exercise 3 A factory is engaged in manufacturing three products, A, B and C which involve lathe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 1 and 1 hours respectively. Similarly, they are 3, 1, 3 hours for one unit of B and 1, 3, 1 hours for one unit of C. The profits per unit of A, B and C are Rs. 20, Rs. 20 and Rs. 30 respectively. Assume that there are 300 hours of lathe time, 300 hours of grinder time and ~~20~~ 240 hours of assembly time. Formulate the problem as an LPP so as to maximize the profit.

Exercise 4 Three different types of lorries A, B and C have been used to transport 60 tons solid and 35 tons liquid substances. A type lorry can carry 7 tons solid and 3 tons liquid, B type lorry can carry 6 tons solid and 2 tons liquid and C type lorry can carry 3 tons solid and 4 tons liquid. The costs of transportation are Rs. 500, Rs 400 and Rs 450 per lorry of A, B and C respectively. Formulate this problem as an LPP so as to minimize the cost.

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## 2.5 Graphical solution of LPP

If the objective function of an LPP is a linear function of two variables, then the problem can be easily solved graphically. In this method, we consider the inequations of the constraints as equations and draw the lines corresponding to these equations in a two dimensional plane and use the nonnegativity restrictions. These lines define the region, in general a region, bounded by a polygon. Sometimes this region may be unbounded. This region, which satisfies all the ~~constrd~~ constraints and non-negativity restrictions, is called the feasible region of the problem. The first step in the graphical method is to draw the feasible region. Now, by trial and error we try to find a point ~~on~~ in the feasible region which gives the optimal value (maximum or minimum) of the objective function according to the problem. We will see afterwards that ~~resp~~ the feasible region will be either a convex polyhedron or a convex polytope and if optimal solution exists, then one of the extreme point of the convex region will give the optimal solution. Hence either the extreme points or corner points of the feasible region need to be considered or the point can be found by translating the straight line given by the objective function for some particular value of $z$, through this region. As the variable are constrained to be non-negative in all LPP, we need only examine the first quadrant or the non-negative of the two dimensional plane in graphical method.

Example 1  Draw the feasible space of the following LPP and solve it graphically :

$$\text{Maximize } Z = 150x + 100y$$

subject to $\cancel{6x + 6y \le 90}$  $6x + 5y \le 60$

$\cancel{15x + 5y \le 90}$  $15x + 6y \le 90$

$\cancel{x \geq 0, y \geq 0}$

Solution: OX and OY are the axes and only the first quadraint is considered as $x \geq 0$, $y \geq 0$.

The equation corresponding to the inequation $6x + 5y \le 60$, is $6x + 5y = 60$

or, $\frac{x}{10} + \frac{y}{12} = 1$

The equation corresponding to the inequations $15x + 6y \le 90$ is $15x + 6y = 90$

or, $\frac{x}{6} + \frac{y}{15} = 1$

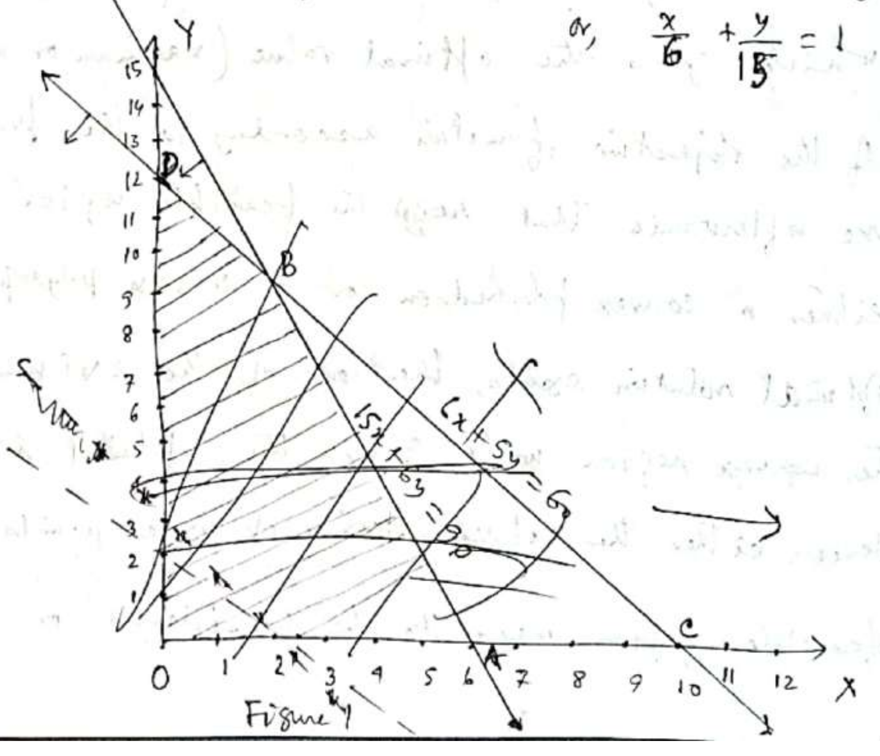


Figure 1