and all the non-basic components are zero. This is the technique to find an initial basic feasible solution to the TP.

Methods for finding initial basic feasible solution

The methods for finding initial basic feasible solution to be discussed here are

(1) North-West Corner method
(2) Row minima method
(3) Column minima method
(4) Matrix minima method
(5) Vogel's Approximation method

Now the above methods are discussed with examples:

(1) North-West Corner method (Rule)

Step 1 Allocate in the cell at the North-West corner, i.e., at (1,1) cell of the table, the maximum amount allowable so that either the capacity of the first row is exhausted or the demand of the first column is satisfied.

Thus $x_{11} = \min(a_1, b_1)$

Step 2 (i) If $b_1 > a_1$, we cross out the first row and adjust the associated amounts of supply and demand by subtracting the allocated amount. Then move down vertically to second row and allocate in (2,1) cell an amount x_{21}, where

$x_{21} = \min(a_2, b_1 - x_{11})$

(ii) If $a_1 > b_1$, we cross out the first column and adjust the associated amounts of supply and demand by subtracting the allocated amount, and then we move right horizontally to the second column and allocate in the $(1,2)$ cell an amount $x_{12} = \min(a_1 - x_{11}, b_2)$

(iii) If $a_1 = b_1$, there is a tie, we cross out both the rows and columns and go to the cell $(2,2)$ and take either $x_{12} = 0$ or $x_{21} = 0$

Step 3 Now allocate in the same way as before to the new matrix and move forward to get a basic feasible solution.

For case (iii) of step 2, we get a degenerate basic feasible solution.

Note: We always make a TP balanced one if it is not, to get always a feasible solution, so that we can get at least one basic feasible solution.

If $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$, then if $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$ then we make the problem a balanced one by introducing a dummy or fictitious destination to the transportation table and requirement at the dummy destination will be assumed to be $\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$ and the transportation from any source to the dummy destination is taken as zero.

If $\sum_{i=1}^{n} a_i < \sum_{j=1}^{n} b_j$ then to make the problem balanced we take a dummy source or origin and allot availability $\sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i$ and the cost from dummy origin to any destination is taken as zero.

Example 1 Find a basic feasible solution by North-West corner rule to the following transportation problem:

Destinations

Origin						
	2	11	10	3	7	4
	1	4	7	2	1	8
	3	9	4	8	12	9
	3	3	4	5	6	

Solution: Here $\sum_{i=1}^{3} a_i = \sum_{j=1}^{5} b_j = 21$, so it is a balanced TP. Now we ~~to~~ find a basic feasible solution by North-West corner method, the steps are shown in the following table:

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	$\boxed{3}$ 2	11	10	3	7	~~4~~ 1
O_2	1	4	7	2	1	8
O_3	3	9	4	8	12	9
b_j	~~3~~	3	4	5	6	

→

	D_2	D_3	D_4	D_5	
O_1	$\boxed{1}$ 11	10	3	7	~~1~~
O_2	4	7	2	1	8
O_3	9	4	8	12	9
	~~3~~ 2	4	5	6	

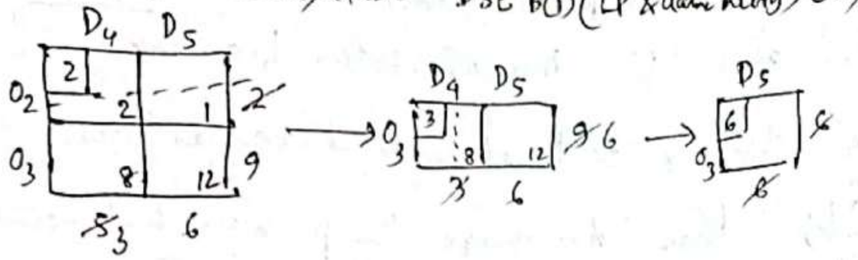
→

	D_2	D_3	D_4	D_5	
O_2	$\boxed{2}$ 4	7	2	1	~~8~~ 6
O_3	9	4	8	12	9
	~~2~~	4	5	6	

→

	D_3	D_4	D_5	
O_2	$\boxed{4}$ 7	2	1	~~6~~ 2
O_3	4	8	12	9
	~~4~~	5	6	

→



So, an initial basic feasible solution by North-West Corner rule is $x_{11} = 3$, $x_{12} = 1$, $x_{22} = 2$, $x_{23} = 4$, $x_{24} = 2$,

$x_{34} = 3$ and $x_{35} = 6$ and

The corresponding cost

$= 3\times2 + 1\times11 + 2\times4 + 4\times7 + 2\times2 + 3\times8 + 6\times12$

$= 6 + 11 + 8 + 28 + 4 + 24 + 72$

$= 153$

Now, also we can write down the whole basic feasible solution in a table with this short-cut approach

	D_1	D_2	D_3	D_4	D_5	
O_1	$\boxed{3}$ 2	$\boxed{1}$ 11	10	3	7	~~4~~ ~~1~~
O_2	1	$\boxed{2}$ 4	$\boxed{4}$ 7	$\boxed{2}$ 2	1	~~8~~ ~~6~~ ~~2~~
O_3	3	9	4	$\boxed{3}$ 8	$\boxed{6}$ 12	~~9~~ ~~6~~
	3	~~3~~ 2	~~4~~	~~5~~ ~~3~~	~~6~~	

Exercise 1 Find an initial basic feasible solution by north-west corner rule for the following transportation problem:

	D_1	D_2	D_3	D_4	
O_1	3	7	2	1	11
O_2	9	4	7	3	20
O_3	10	2	8	3	35
	10	5	21	30	

Is the solution degenerate?