(2) Row minima method : In this method, we first consider the first row and find the minimum cost cell. Let $(1, k)$ be the cell in the first row with minimum cost. We allot in this cell the maximum allocation $x_{1k} = \min(a_1, b_k)$. If $a_1 < b_k$ then $x_{1k} = a_1$ and we cross out the first row and consider the remaining table and proceed in this way. Again if $a_1 > b_k$, then $x_{1k} = b_k$ and we cross out the kth column and consider the remaining table and proceed next in this way. If $a_1 = b_k$, then both first row and kth column is crossed out and we proceed with the remaining table in the same way.

Example 2 Find an initial basic feasible solution by row minima method for the following transportation problem :

	D_1	D_2	D_3	D_4	D_5	
O_1	3	11	10	3	7	4
O_2	1	9	7	2	1	8
O_3	3	9	4	8	12	9
	3	3	4	5	6	

Solution : we find an initial basic feasible solution by row minimum method as shown in the following compact table :

	D_1	D_2	D_3	D_4	D_5	
O_1	[3] 3	11	10	[1] 3	7	~~4~~ ~~1~~
O_2	1	9	7	[2] 2	[6] 1	~~8~~ ~~2~~
O_3	3	[3] 9	[4] 4	[2] 8	12	~~9~~ ~~5~~ ~~2~~
	~~3~~	~~3~~	~~4~~	~~5~~ ~~4~~ ~~2~~	~~6~~	

So, a basic feasible solution by row minima method is

$x_{11} = 3$, $x_{14} = 1$, $x_{24} = 2$, $x_{25} = 6$, $x_{32} = 3$, $x_{33} = 4$, $x_{34} = 2$

It is a non-degenerate basic feasible solution as the number of non-zero component is $7 = (5+3-1)$

The cost corresponding to this solution is 78 units.

(3) Column minima method: It is exactly similar to the row minima method, the only difference is that instead of rows we start with columns. So, here we did not give any example. Try this method with previous example as exercise.

(4) Matrix minima method (least cost entry method)

This method finds a better starting solution. In this method we first find out the cell with minimum cost in the cost matrix and allocate in that the maximum allowable amount. We then cross out the satisfied row or column or both and adjust amount of supply and demand accordingly. Then we repeat the same process with the new matrix and so on.

Example 3. Apply matrix minima method to find an initial basic feasible solution to the following transportation problem:

O_1	2	11	10	3	7	4
O_2	1	4	7	2	1	8
O_3	3	9	4	8	12	9
	3	3	4	5	6	

Solution: we find an initial basic feasible solution by matrix minima method as shown in the following compact table:

	D_1	D_2	D_3	D_4	D_5	
O_1	2	11	10	$\boxed{4}$ 3	7	~~4~~
O_2	$\boxed{3}$ 1	4	2	2	$\boxed{5}$ 1	~~8~~ ~~5~~
O_3	3	$\boxed{3}$ 9	$\boxed{4}$ 4	$\boxed{1}$ 8	$\boxed{1}$ 12	~~9~~ ~~5~~ ~~4~~ 1
	~~3~~	~~3~~	~~4~~	~~1~~ ~~5~~	~~6~~ 1	

(In the cost matrix cell (2,1) and (2,5) have the minimum cost. So, we select any one of these. Let us select (2,1) cell and allocate $x_{21} = \min\{a_2, b_1\} = \min(8,3) = 3$)

So, a ~~basic~~ basic feasible solution by Matrix minima method is $x_{14} = 4$, $x_{21} = 3$, $x_{25} = 5$, $x_{32} = 3$, $x_{33} = 4$, $x_{34} = 1$, $x_{35} = 1$

and the corresponding cost = 83 units

(3) Vogel's Approximation Method (VAM) or Unit penalty method.

VAM is an improved version of the matrix minima method and produces generally a better starting solution in respect of cost minimization.

The steps to be followed in this method are:

<u>Step 1</u> Find out for each row and each column the smallest unit cost and next smallest unit cost (If there are two cells in a row or in a column with same smallest unit cost, then the next smallest unit cost will be the same as the 'smallest unit cost'). Then determine a ~~penal~~ penalty measure by taking the difference of these two and displays these differences in parenthesis in

the transportation table by the sides of the availabilities in case of rows and below the requirements in the case of columns.

<u>Step 2</u> Identify the row or column with largest penalty (if there is tie, take any one of them) and in this selected row or column, allocate the maximum allowable amount to the least cost cell. Adjust the supply and demand and cross out the satisfied row or column. If a row and a column be satisfied simultaneously, then cross out both of them (In this case, we always get a degenerate basic feasible solution) and we have to choose a basic cell with basic component zero, so that no loop is contained in the basic cells.)

The we go to the next table and repeat the same process and so on. In this way we get a basic feasible solution by VAM

<u>Example 4</u> Use VAM to obtain an initial basic feasible solution to the following problem:

	D_1	D_2	D_3	D_4	D_5	
O_1	2	11	10	3	7	4
O_2	1	4	7	2	1	8
O_3	3	9	4	8	12	9
	3	3	4	5	6	

Solution: To find an initial basic feasible solution by VAM, we show the whole computation in a