single table in the following manner :

	D_1	D_2	D_3	D_4	D_5						
O_1	2	11	10	$\boxed{4}$ 3	7	4 (1)	4 (1)	4 (1)	4 (1)	4 (8)	
O_2	1	$\boxed{2}$ 4	7	2	$\boxed{6}$ 1	8 (0)	2 (1)				
O_3	$\boxed{3}$ 3	$\boxed{1}$ 9	$\boxed{4}$ 4	$\boxed{1}$ 8	12	9 (1)	9 (1)	9 (1)	5 (5)	2 (1)	2
	3 (1)	3 (5)	4 (3)	5 (1)	6 (6)						
	3 (1)	3 (5)	4 (3)	5 (1)							
	3 (1)	1 (2)	4 (6)	5 (5)							
	3 (1)	1 (2)		5 (5)							
		1 (2)		5 (5)							
		1		1							

Number of occupied cell is $7 = (5+3-1)$ and so this gives a non-degenerate basic feasible solution by VAM which is

$x_{14} = 4,\ x_{22} = 2,\ x_{25} = 6,\ x_{31} = 3,\ x_{32} = 1,\ x_{33} = 4,\ x_{34} = 1$

and the corresponding cost $= 64$ unit.

Note : We see that, VAM gives the best initial basic feasible solution in respect of cost as as we have used the same problem, in calculating initial basic feasible solution in the above examples.

Now we want to check whether a basic feasible solution is optimal or not. Now we state the procedure for checking the optimality of a basic feasible solution of a TP.

SOLUTION OF A TRANSPORTATION PROBLEM

~~OPTIMALITY CONDITION~~

We always consider a balanced TP (If it is not, we make it balanced by introducing dummy rows or columns)

Step 1 We first find out a basic feasible solution of the ~~given~~ balanced TP by any one of the method discussed earlier (The best method to find an initial basic feasible solution is VAM)

Initial basic feasible ~~solution~~ components are shown in the upper left corner of the cell. There will be basic $(m+n-1)$ cells for non-degenerate basic feasible solution. For degenerate basic feasible solution there will less than $(m+n-1)$ basic cells. So, we have to choose that number of basic cells by which the basic components are less from $(m+n-1)$ in such a way that together with those earlier basic cells they do not contain any loop. So, for those chosen cells, the basic components are zero. So these $(m+n-1)$ components gives a degenerate basic feasible solution. Now for those cells in the basic components where zeros are taken for the variables, put $\varepsilon > 0$ (a very small positive number) for every zero and then proceed to the next step for both non-degenerate and degenerate solution.

Step 2 Determine a set of $(m+n)$ numbers u_i and v_j, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ such that for all occupied (basic) cells, $c_{ij} = u_i + v_j$. ... (1) ~~So they are more~~ As they are $(m+n)$ numbers, and the number of equations are $m+n-1$, so we can choose one value arbitrarily. We put that u_i or v_j for which the corresponding row or column ~~cell~~ contains maximum number of occupied cells is put to zero. Then we we find all u_i, v_j from (1)

Step 3 Now we calculate $\Delta_{ij} = c_{ij} - (u_i + v_j)$ for all unoccupied (non-basic) cells and put it in the middle of that cell within a circle.

Case 1 If all $\Delta_{ij} \geq 0$ for unoccupied cells, then the ~~solution~~ basic feasible solution is optimal. The optimal solution is unique if $\Delta_{ij} > 0$ for all unoccupied cells. But the optimal solution is not unique if at least one $\Delta_{ij} = 0$ for unoccupied cell.

Case 2. If at least one $\Delta_{ij} < 0$, then the basic feasible solution is not optimal and we go to step 4

Step 4 Now to find a new basic feasible solution, we choose that (r, s) cell for which Δ_{rs} is negative and it is the least negative of all $\Delta_{ij} < 0$ we form a loop containing the (r, s) cell and those basic cells which will be needed for this. Such loop always exists. Starting from this (r, s) cell, allocate an amount θ with alternative positive and negative signs to all the end points of the closed loop so that supply and demand constraints are always satisfied. We choose θ as the minimum quantity for which all other allocations made will be non-negative and one of them will be zero and discarded from the basic cell and (r, s) is the new basic cell with allocation θ.

Step 5 Repeat the steps 2 to step 4 until all the evaluation (Δ_{ij}) become non-negative.

Example 5 Obtain an optimum basic feasible solution to the following transportation problem:

	D_1	D_2	D_3	D_4	
O_1	19	14	23	11	11
O_2	15	16	12	21	13
O_3	30	25	16	39	18
	6	10	11	15	

Solution: In this problem $\sum a_i = \sum b_j = 40$, so it is a balanced transportation problem.

We first determine an initial basic feasible solution to this problem using VAM as shown in Table 1

Table 1

	D_1	D_2	D_3	D_4					
O_1	19	14	23	$\boxed{11}$ 11	11 (3)				
O_2	$\boxed{6}$ 15	$\boxed{3}$ 16	12	$\boxed{4}$ 21	13 (3)	13 (3)	9 (3)	3 (4)	3
O_3	30	$\boxed{7}$ 25	$\boxed{11}$ 16	39	18 (9)	18 (9)	18 (9)	18 (9)	7
	6 (4)	10 (2)	11 (4)	15 (10)					
	6 (15)	10 (9)	11 (4)	4 (18)					
	6 (15)	10 (9)	11 (4)						
		10 (9)	11 (4)						
		10							

Number of basic cells is $6\ (= 4+3-1)$. So, the initial basic feasible solution is a non-degenerate ~~bfs~~ basic feasible solution

~~we shall now find out the Δ_{ij}~~

we now find out u_i and v_j, $i = 1, 2, 3$, $j = 1, 2, 3, 4$, using $c_{ij} = u_i + v_j$ for basic cells.

putting $u_2 = 0$ and calculate all $\Delta_{ij} = c_{ij} - (u_i + v_j)$ for non-basic cells as shown in Table 2

Table - 2

	D_1	D_2	D_3	D_4	u_i
O_1	14 ; 19	8 ; 14	26 ; 23	$\boxed{11}$; 11	−10
O_2	$\boxed{6}$; 15	$\boxed{3}$; 16	5 ; 12	$\boxed{4}$; 21	0
O_3	6 ; 30	$\boxed{7}$; 25	$\boxed{11}$; 16	9 ; 39	9
v_j	15	16	7	21	

So, $\Delta_{ij} = c_{ij} - (u_i + v_j) \geq 0$ for all non-basic cells. So, optimality is satisfied and the optimal solution is unique as no $\Delta_{ij} = 0$ for a non-basic cell. So, the optimal solution is $x_{14} = 11$, $x_{21} = 6$, $x_{22} = 3$, $x_{24} = 4$,

$x_{32} = 7$, $x_{33} = 11$.

and the minimum cost.

$= 11 \times 11 + 6 \times 15 + 3 \times 16 + 7 \times 25 + 11 \times 16$

$= 694$

Example 6 Solve the following transportation problem by finding an initial basic feasible solution by matrix minima method :