value of $x_{ij} = 1$ and other values of x_{ij} are zero.

Algorithm for the Hungarian method

<u>Step 1</u> In ~~row~~ each row of the cost matrix, locate the smallest cost and subtract it from every element in that row. From the resulting matrix, identify the smallest element of each column and subtract it from each ~~column~~ element ~~of the column~~ in that column.

The revised cost matrix will have at least one zero in every row and every column. If there is one zero in any row or any column, that zero is put with $\square$ for identification and is called assigned zero.

<u>Step 2</u> If the revised matrix has n zero entries only and no two of which are in the same row or column, then the assignment at those assigned zero cells is optimal. Otherwise follow step 3

<u>Step 3</u> If at least one row or column of the revised cost matrix contains more than one zero entries then draw the minimum number of

horizontal and vertical lines to cover all zeroes in the revised cost matrix. Each horizontal line must cover entire row and similarly for each vertical ~~lr~~ line.

Now two cases may arise:

(a) The minimum number of lines so drawn is equal to the order of the cost matrix and in that, optimal assignment is possible and we will follow step 4.

(b) The minimum number of lines so drawn be less than the order of the cost matrix and then we follow step 5.

<u>Step 4</u> Examine the rows ~~and~~ of the revised matrix starting from the first row, if any row contains only one zero, there will be made an assignment and put this zero within $\square$.

When all the rows are checked, draw vertical lines along those columns ~~containg~~ containing assigned zeroes.

Then follow the same procedure for ~~column uncrossed~~ uncrossed columns one after another.

If there be only one zero in any column, an assignment is to be made at that zero and put the zero with $\square$ as before. When all uncrossed columns are checked draw horizontal lines along those rows which contains assigned zeroes. Finally, we get all assigned zeroes by allocating arbitrarily in the position of zeroes and not covered by the lines.

If there remains more than one zero in a row or in a column uncovered, we make assignment for these zeroes by trial and error method and in this case alternative solution may exist. It is to be noted that there will ~~be~~ be only one assigned zero in every row and every column for optimal solution and the assignment will be made at these assigned zero cells only.

<u>Step 5</u> ~~If~~ Select the smallest uncovered element in the above cost matrix. Subtract the element from every uncovered element and add it to every element at the intersection of two lines and other elements remain unchanged. Then go to step 3 with modified cost matrix. Repeat the steps 5 and 3 if needed and apply

step 4.

- Unbalanced assignment problem

If in any assignment problem the number of jobs and the number of workers are not equal, it is known as unbalanced assignment problem. This unbalanced problem can be converted to a balanced problem by just adding a fictitious job or worker, whichever has the deficiency, with zero cost in the corresponding row or column in the cost matrix. Then assignment algorithm, is applied to the balanced problem.

Example 1 Find the optimal assignment for the following assignment problem and the minimum cost.

	M_1	M_2	M_3	M_4	M_5
J_1	3	8	2	10	3
J_2	8	7	2	9	7
J_3	6	4	2	7	5
J_4	8	4	2	3	5
J_5	9	10	6	9	10

Solution: This is a balanced assignment problem. We first subtract the minimum cost of the row from every element of that row and do it for all rows.

Table 1

	M_1	M_2	M_3	M_4	M_5
J_1	1	6	0	8	1
J_2	6	5	0	7	5
J_3	4	2	0	5	3
J_4	6	2	0	1	3
J_5	3	4	0	3	4

Next, we subtract minimum element of the column from all elements of that column and do it for all columns.

Table 2

	M_1	M_2	M_3	M_4	M_5
J_1	0	4	0	7	0
J_2	5	3	0	6	4
J_3	3	0	0	4	2
J_4	5	0	0	0	2
J_5	2	2	0	2	3

Next we draw minimum number of horizontal and vertical lines to cover all the zeros.

Table 3

0	4	0	7	0
5	3	0	6	4
3	0	0	4	2
5	0	0	0	2
2	2	0	2	3

As the number of lines covering zeroes = 4 < 5 = order of the cost matrix, optimal condition is not satisfied.

The smallest uncovered element in Table 3 is 2. Subtract 2 from each uncovered element. Add 2 to the elements at the intersection of two lines, we get the new matrix as given in Table 4

Table 4

0	4	2	7	0
3	4	0	4	2
3	0	2	4	2
5	0	2	9	2
0	0	0	0	1

As the number of lines covering zeros = 5 = order of the cost matrix, so the optimal condition is satisfied. So, we make the optimal assignment in the table 5 as follows.

$\cancel{0}$				$\boxed{0}$
		$\boxed{0}$		
	$\boxed{0}$			
	$\cancel{0}$		$\boxed{0}$	
$\boxed{0}$	$\cancel{0}$	0	0	

So, the optimal assignment is given by

$J_1 \longrightarrow M_5$

$J_2 \rightarrow M_3$

$J_3 \rightarrow M_2$

$J_4 \rightarrow M_4$

$J_5 \rightarrow M_1$

And the minimum cost is $= 3+2+4+3+9 = 21$