Here in figure 1, OX and OY are the axes and AB and CD are the lines ~~3x+2y~~ $15x+6y=90$ and $6x+5y=60$ respectively. Since $x \geq 0, y \geq 0$, the feasible region (as shown by the shaded region in figure 1) is the region bounded by polygon OABD. Here the extreme points or corner points of the feasible region is $O(0,0)$, $A(6,0)$, $B\left(\frac{30}{13}, \frac{120}{3}\right)$ and $D(0,12)$ (O, A and D are easy to find out and solve $6x+5y=60$ and $15x+6y=90$ to set B)

For any particular value of Z, the graph of the ~~obj~~ objective function regarded as an equation is a straight line and as Z varies, a family of parallel lines is generated. A few of these lines are graphed for specific values of Z and are shown in figure 1 as shown by dotted lines.

For $Z=300$, the objective function is ~~2x+3y=~~ $3x+2y=6$

For $Z=450$, the objective function is $3x+2y=9$

For $Z=600$, the objective function is $3x+2y=12$

For $Z=900$, the objective function is $3x+2y=18$

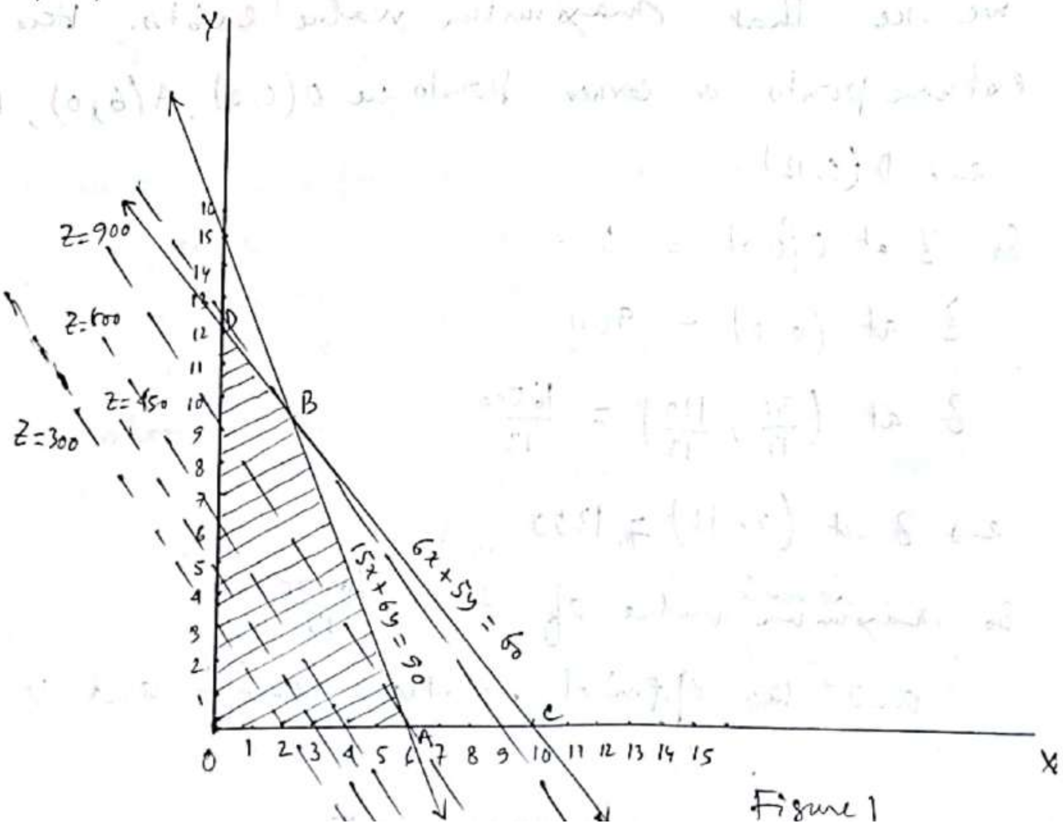


Figure 1

So, from the graph of the objective function, which gives the profit here, we see that profit increases as the profit line or the objective function line is ~~trasl~~ translated away from the origin. So, our aim is to translate the ~~from~~ objective function away from the origin in such a way that it contains only one point of the feasible region. Here we see that B is such a point where the objective function passes through only one point B of the feasible region.

So, we get the maximum value of the objective function is at $B\left(\frac{30}{13}, \frac{120}{13}\right)$. So, the maximum value of the objective function is $150 \times \frac{30}{13} + 100 \times \frac{120}{13} = \frac{4500+12000}{13}$

$$= \frac{16500}{13}$$

and an optimal solution is $x = \frac{30}{13}$, $y = \frac{120}{13}$

Alternatively, we can also find in the following way the optimal solution: By moving the objective function, we see that maximum value exists. Here the extreme points or corner points are $O(0,0)$, $A(6,0)$, $B\left(\frac{30}{13}, \frac{120}{13}\right)$ and $D(0,12)$

So, Z at $O(0,0) = 0$

Z at $(6,0) = 900$

Z at $\left(\frac{30}{13}, \frac{120}{13}\right) = \frac{16500}{13}$

and Z at $(0,12) = 1200$

So maximum value of $Z = \frac{16500}{13}$

and the optimal solution $x = \frac{30}{13}$ and $y = \frac{120}{13}$

Example 2 Solve the following LPP graphically :

Maximise $Z = 2x_1 + x_2$

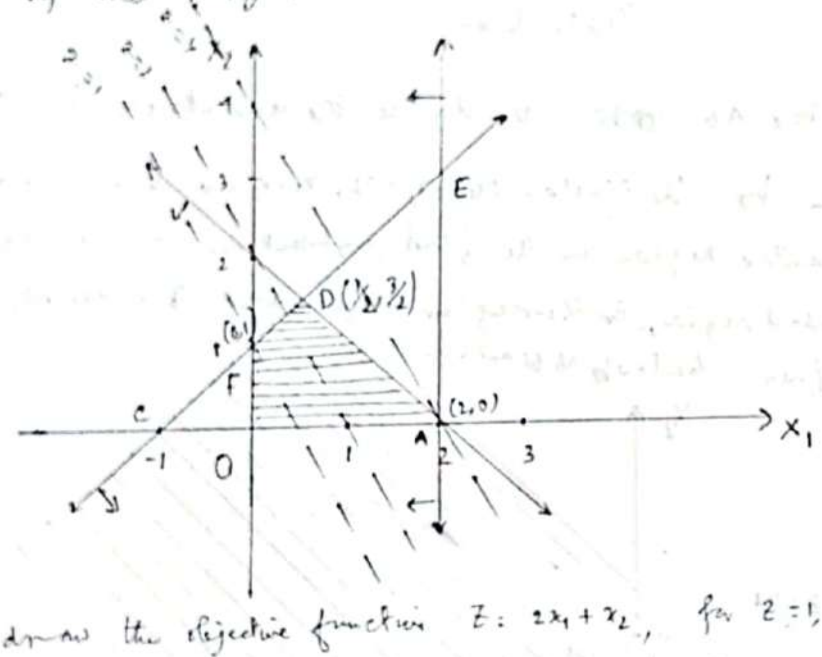
subject to $x_1 + x_2 \leq 2$

$-x_1 + x_2 \leq 1$

$x_1 \leq 2$

$x_1 \geq 0,\ x_2 \geq 0$

Solution: (a) Ox_1 and Ox_2 are the axes and AB, CD and AE represent the straight lines $x_1 + x_2 = 2$, $-x_1 + x_2 = 1$ and $x_1 = 2$. Also $x_1 \geq 0$ and $x_2 \geq 0$. So, the feasible region is in the first quadrant and the feasible region ~~is~~ (shown by the shaded region) is region bounded by the polygon OADF



We draw the objective function $Z = 2x_1 + x_2$, for $Z = 1, 2$ and 4

For, $Z = 1$, it is the line $2x_1 + x_2 = 1$

For, $Z = 2$, it is the line $2x_1 + x_2 = 2$

For, $Z = 4$, it is the line $2x_1 + x_2 = 4$

They are shown by dotted lines. As these passes away from the origin, the value of the objective functions increases and as the feasible region is bounded after some time it passes away from the feasible region. So, maximum value exists. Here the corner points or extreme points of the feasible region is $O(0,0)$, $A(2,0)$, $D(\frac{1}{2}, \frac{3}{2})$ and $F(0,1)$

Z at $O(0,0) = 0$

Z at $A(2,0) = 4$

Z at $D(\frac{1}{2}, \frac{3}{2}) = 1 + \frac{3}{2} = \frac{5}{2}$

Z at $F(0,1) = 1$

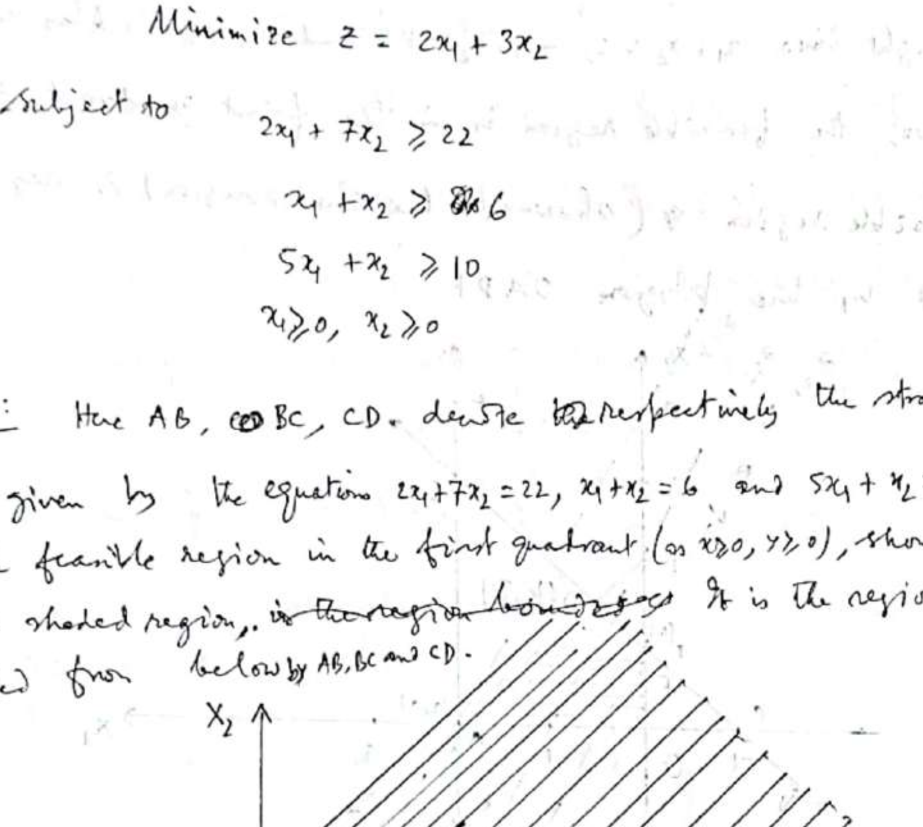
So maximum value of $Z = 4$ and an optimal solution is $x_1 = 2,\ x_2 = 0$.

Example 3 Solve graphically, the following LPP:

$$\text{Minimize } Z = 2x_1 + 3x_2$$

Subject to
$$2x_1 + 7x_2 \geq 22$$
$$x_1 + x_2 \geq 6$$
$$5x_1 + x_2 \geq 10$$
$$x_1 \geq 0,\ x_2 \geq 0$$

Solution: Here AB, BC, CD denote respectively the straight lines given by the equations $2x_1+7x_2=22$, $x_1+x_2=6$ and $5x_1+x_2=10$. So, the feasible region in the first quadrant (as $x\geq 0, y\geq 0$), shown by the shaded region, ~~is the region bounded~~ It is the region bounded from below by AB, BC and CD.



we draw the objective function $Z = 2x_1+3x_2$ for some values of $Z = 24, 12, 6$ by dotted lines. For $Z = 24$, the line is $\frac{x_1}{12} + \frac{x_2}{8} = 1$

For $Z = 12$, the line is $\frac{x_1}{6} + \frac{x_2}{4} = 1$

and for $Z = 6$, the line is $\frac{x_1}{3} + \frac{x_2}{2} = 1$.

So, as these goes towards the origin, the values of Z decreases and after some times, it goes out side the feasible region, so