minimum value of Z exists. Here the extreme points of the unbounded region are $A(11,0)$, $B(4,2)$, $C(1,5)$ and $D(0,10)$.

Now Z at $A(11,0) = 2\times 11 + 3\times 0 = 22$

Z at $B(4,2) = 2\times 4 + 3\times 2 = 14$

Z at $C(1,5) = 2\times 1 + 3\times 5 = 17$

Z at $D(0,10) = 2\times 0 + 3\times 10 = 30$

So, minimum value of $Z = 14$ and an optimal solution is $x_1 = 4,\ x_2 = 2$

Note: If $Z = 2x_1 + 3x_2$ and the problem is of maximization for the previous problem (i.e. Example 3), then we can make the objective ~~larger~~ function value as large as possible, moving away from the origin, as it will always be in the feasible region. Then the problem has no finite maximum. So, the problem has unbounded solution.

<u>Exercises</u> 1. Solve graphically the following LPP:

Minimize $Z = 3x + 5y$

subject to $2x + 3y \geq 12$

$-x + y \leq 3$

$x \leq 4$

and $y \geq 3$

$x \geq 0,\ y \geq 0$

2. Solve graphically the following LPP:

Minimize $Z = -2x_1 + x_2$

Subject to $x_1 + x_2 \geq 6$

$3x_1 + 2x_2 \geq 16$

$x_2 \leq 9$

$x_1 \geq 0,\ x_2 \geq 0$

3. Solve graphically the following LPP :

Maximize $Z = 2x_1 + 5x_2$

subject to $5x_1 + 6x_2 \geq 30$

$3x_1 + 2x_2 \leq 21$

$x_1 + x_2 \leq 12$

$x_1 \geq 0,\ x_2 \geq 0$

4. ~~Max~~ Solve ~~the~~ graphically the following LPP

Maximize $Z = 3x_1 - x_2$

subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 2$

$x_2 \leq 4$

~~and~~ $x_1 \geq 0,\ x_2 \geq 0$.

2.6 Basic solution

Consider a system of linear equations $Ax = b$ where $A = [a_{ij}]_{m \times n}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ $b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$ and $m < n$.

So, it is a system of m linear equations in n unknowns $(m < n)$. Let rank of $A = r(A) = m$. If any $m \times m$ non-singular be arbitrarily selected from A and if we assume all $(n-m)$ variable zero, which are not associated with the column vectors of the non-singular matrix, the solution so obtained is called a basic solution. That is, let $x_B = \begin{bmatrix} x_{B_1} \\ x_{B_2} \\ \vdots \\ x_{B_m} \end{bmatrix}$ be the column vectors of those variables which are related to the column vectors of the chosen non-singular matrix B which is of order m. Now Solve $Bx_B = b$ and put other $(n-m)$ variables ~~to~~ the values zero. Then the first m components and the other $(n-m)$ components with values zero constitutes a solution of $Ax = b$. This solution is called a basic solution of $Ax=b$. As you can choose m columns from n ~~of~~ columns in ${}^nC_m$ ways, so there are atmost ${}^nC_m$ basic solution.

A solution ~~x~~ $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ of $Ax=b$ is said to be a ~~feas~~ feasible solution if $x_i \geq 0$ for $i=1,2,\ldots,n$

So, a basic solution is said to be a basic feasible solution if $x_{B_i} \geq 0$ $i=1,2,\ldots,m$ because other $(n-m)$ components are zero. $x_{B_1}, x_{B_2}, \ldots, x_{B_m}$ are called basic variables and other $(n-m)$ variables are called non-basic variables.

If all the basic components of a basic solution be ~~non-zero~~ non-zero, then it is called a non-degenerate basic solution, ~~so~~ otherwise it is called a degenerate basic solution.

We find basic solutions of some problems:

<u>Example 1</u> Find the basic solution or solutions, if there be any, of the set of equations

$$2x_1 + 4x_2 - 2x_3 = 10$$
$$10x_1 + 3x_2 + 7x_3 = 33$$

Solution: The set of equations can be written as

$$Ax = b$$

where $A = \begin{bmatrix} 2 & 4 & -2 \\ 10 & 3 & 7 \end{bmatrix} = [a_1, a_2, a_3]$ where

$a_1 = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$, $a_2 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and $a_3 = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$,

$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $b = \begin{pmatrix} 10 \\ 33 \end{pmatrix}$

Here rank of $A = r(A) = 2$ as $\begin{vmatrix} 2 & 4 \\ 10 & 3 \end{vmatrix} = 6 - 40 = -34 \neq 0$

Here the three submatrices of A are

$B_1 = (a_1, a_2) = \begin{bmatrix} 2 & 4 \\ 10 & 3 \end{bmatrix}$. Here $\det B_1 = \begin{vmatrix} 2 & 4 \\ 10 & 3 \end{vmatrix} = -34 \neq 0$

$B_2 = (a_1, a_3) = \begin{bmatrix} 2 & -2 \\ 10 & 7 \end{bmatrix}$. Here $\det B_2 = \begin{vmatrix} 2 & -2 \\ 10 & 7 \end{vmatrix} = 34 \neq 0$

and $B_3 = (a_2, a_3) = \begin{bmatrix} 4 & -2 \\ 3 & 7 \end{bmatrix}$. Here $\det B_3 = \begin{vmatrix} 4 & -2 \\ 3 & 7 \end{vmatrix} = 34 \neq 0$.

So, all three square submatrices are non singular

For the matrix B_1, the basic components are x_1, x_2 and non-basic component is $x_3 = 0$.

now $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = B_1^{-1} b = -\frac{1}{34} \begin{bmatrix} 3 & -4 \\ -10 & 2 \end{bmatrix} \begin{bmatrix} 10 \\ 33 \end{bmatrix}$

$$= -\frac{1}{34} \begin{bmatrix} -102 \\ -34 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

So, $x_1 = 3$, $x_2 = 1$ and $x_3 = 0$ is a basic solution

As $x_i \geq 0$, $i = 1, 2, 3$, so it is also a basic feasible solution