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PHYSICS
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Maxwell Equation, Displacement current, Equation of Continuity, Ampere circuital law: Maxwell Modification

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FOUR MAXWELL EQUATION

Name	Differential Form
Gauss' Law	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$
Gauss' Law for Magnetism	$\nabla \cdot \mathbf{B} = 0$
Faraday' Law	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampere-Maxwell Law	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

EXPLANATION OF FOUR EQUATIONS

1. Gauss' Law or Maxwell's first equation

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

Electric charges produce an electric field. The electric flux across a closed surface is proportional to the charge enclosed.

2. Gauss' Law for Magnetism or Maxwell's second equation

$$\nabla \cdot \mathbf{B} = 0$$

There are no magnetic monopoles. The magnetic flux-and-faradays-law-quantitative across a closed surface is zero.

3. Faraday's Law or Maxwell's third equation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Time-varying magnetic fields produce an electric field.

4. Ampere's Law or Maxwell's fourth equation

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

Steady currents and time-varying electric fields (the latter due to Maxwell's correction) produce a magnetic field.

FOUR MAXWELL EQUATION IN INTEGRAL FORM

Maxwell's Equations

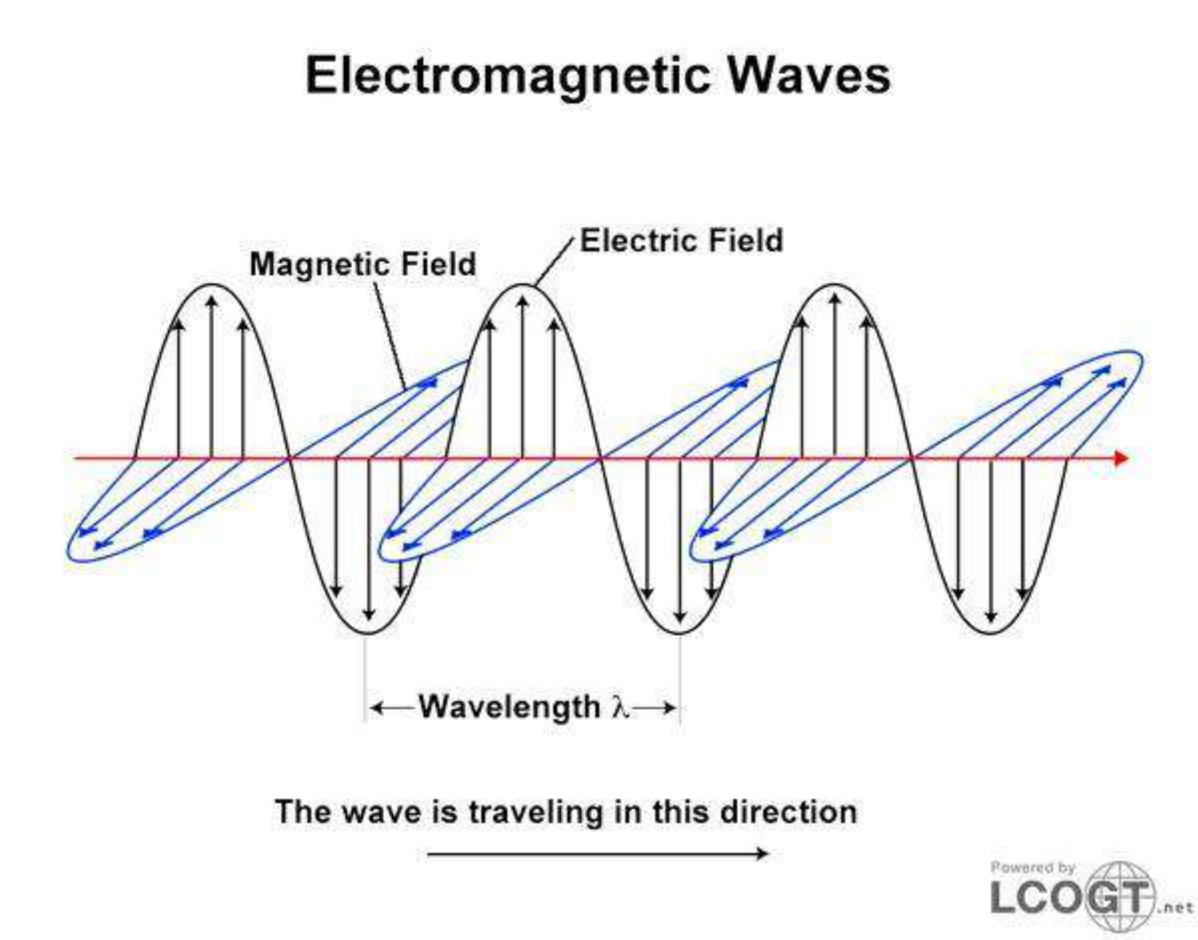
- $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$
- $\oint \mathbf{B} \cdot d\mathbf{A} = 0$
- $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_e}{dt}$
- $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_m}{dt}$

- Maxwell showed that these equations explain the existence of the waves that Hertz discovered.
- The speed of these waves would be $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$
- Using $\mu_0 = 4\pi \times 10^{-7}$ N/A^2 and $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/(Nm^2), find the speed of these waves.

LIGHT IS AN ELECTROMAGNETIC WAVE

Using these equations only we got a relation between the speed of light (an electromagnetic wave) and permeability and permittivity of free space. So now you know why the speed of light depends on the medium it is traveling in.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$



WHAT IS DISPLACEMENT CURRENT

Displacement current comes into play due to the electric field and electric flux which is changing with time.

- **Displacement current is zero unless there is a changing electric field**

$$I_d \equiv \varepsilon_0 \frac{d\Phi_E}{dt}$$

DIFFERENCE BETWEEN CONDUCTION AND DISPLACEMENT CURRENT

1. Conduction current obeys ohm's law as $i = \frac{V}{R}$ but displacement current does not obey ohm's law.

2. Conduction current density is represented by $\vec{J}_c = \sigma \vec{E}$ whereas displacement current density is given by $\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t}$.

3. Conduction current is the actual current whereas displacement current is the apparent current produced by time varying electric field.

Ratio of conduction Current Density $\mathbf{J_c}$ to Displacement Current Density $\mathbf{J_d}$

$$\bar{J}_t = \bar{J}_c + \bar{J}_d$$

$$\bar{J}_c = \sigma \bar{E} \qquad \bar{J}_d = \frac{\partial \bar{D}}{\partial t} = j\omega\varepsilon \bar{E}$$

$$\bar{J}_t = \sigma \bar{E} + j\omega\varepsilon \bar{E}$$

$$\frac{|\bar{J}_c|}{|\bar{J}_d|} = \frac{\sigma}{\omega\varepsilon} = \tan\delta$$

So as ω frequency increases displacement current J_d becomes equally important

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EQUATION OF CONTINUITY : DERIVATION

Equation of Continuity

Volume V bounded by Surface S

$$\frac{dq}{dt} = i = \int_S \vec{J} \cdot d\vec{s} \quad \text{—(1)}$$

$$i = \oint \vec{J} \cdot ds = -\frac{dq}{dt} = -\int_V \left(\frac{\partial \rho}{\partial t}\right) dV$$

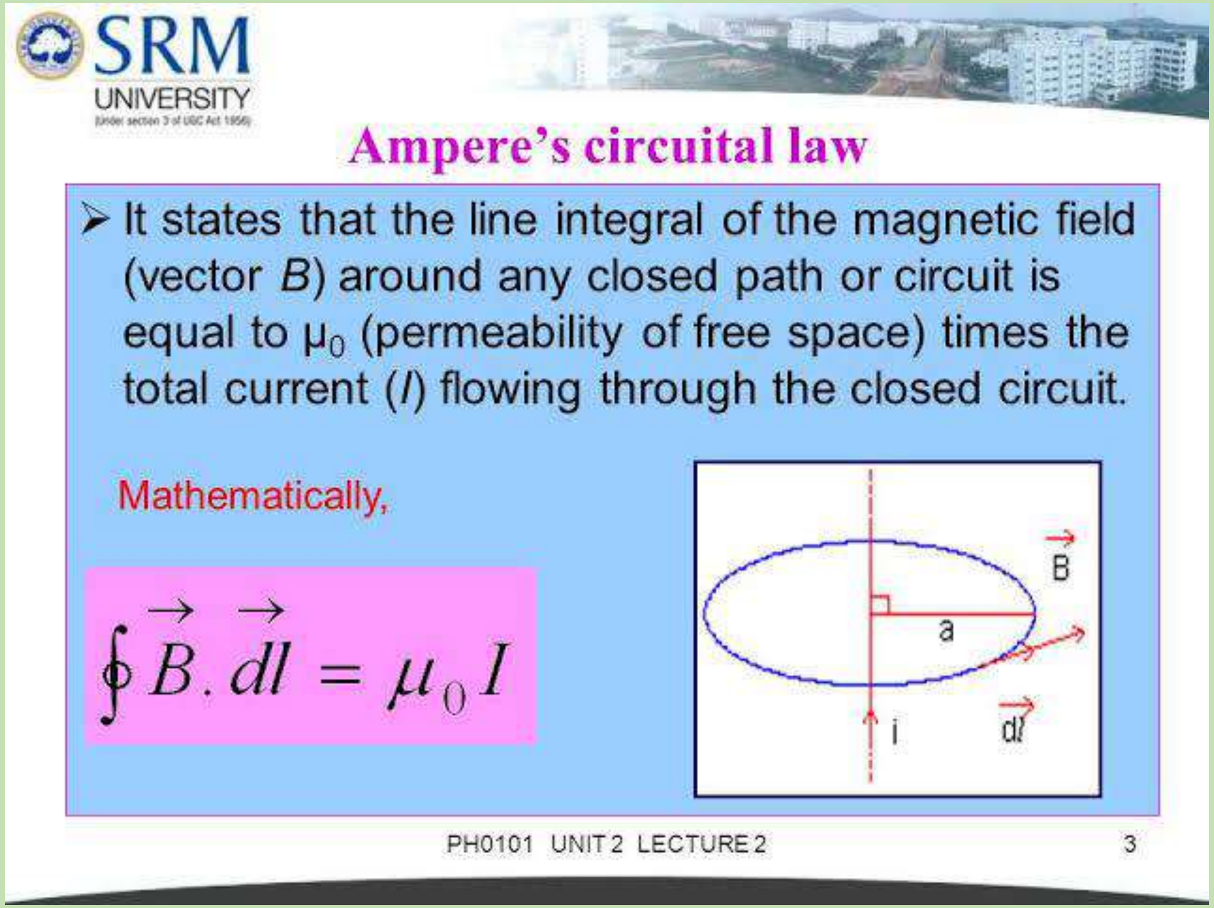
$$\boxed{q = \int_V \rho \, dv}$$

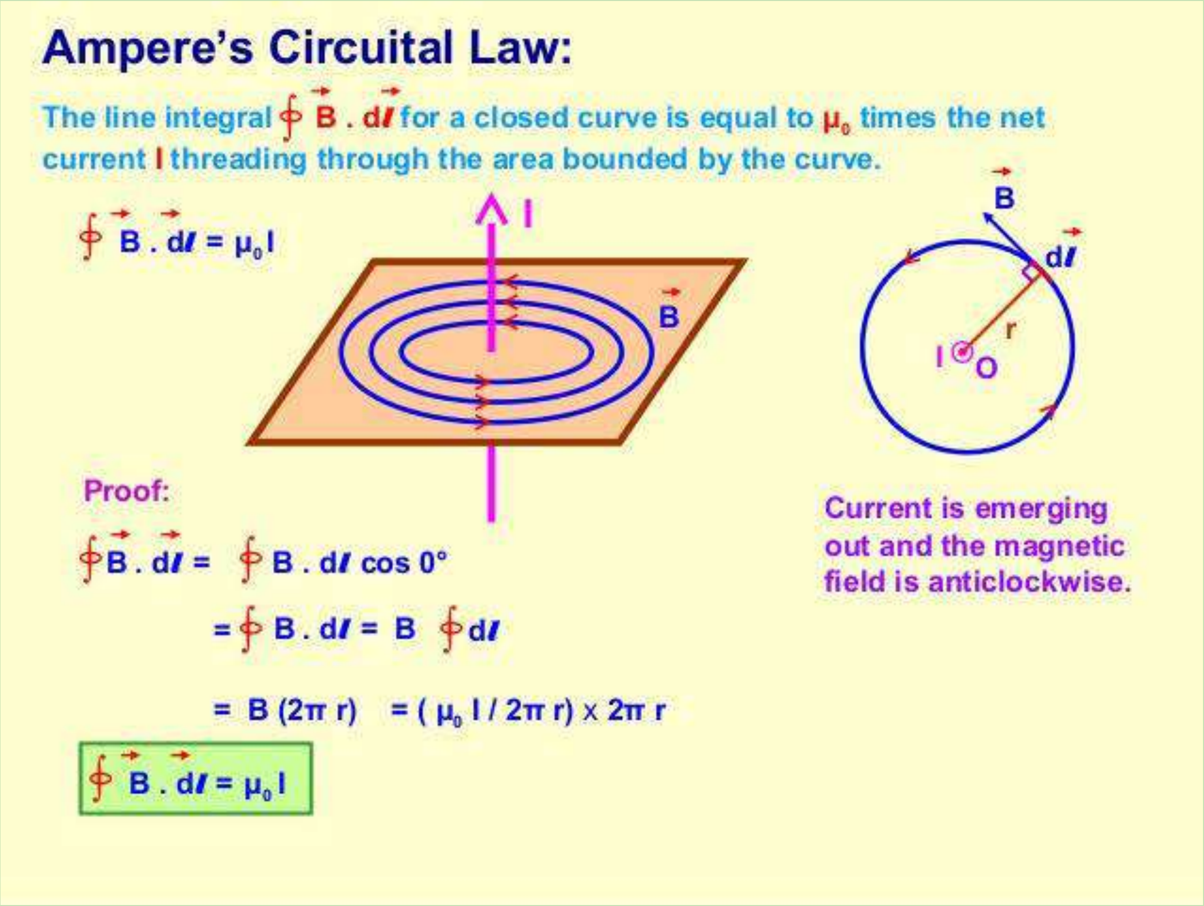
$$\oint_S \vec{J} \cdot d\vec{s} + \int_V \frac{\partial \rho}{\partial t} dv = 0$$

$$\oint \vec{J} \cdot ds = \int_V \vec{\nabla} \cdot \vec{J} \, dV$$

$$\int_V \left(\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t}\right) dV = 0 \qquad \nabla \cdot \text{—}$$

AMPERE CIRCUITAL LAW





AMPERE'S CIRCUITAL LAW: MAXWELL 'S MODIFICATION

The Ampere's circuital law states that the path integration of magnetic field **B** around any closed path equal to the total current through the surface enclosed by that path times μ_0. The integral form is

$$\oint \mathbf{B}.d\mathbf{l} = \mu_0 \iint \mathbf{J} \cdot d\mathbf{l} = \mu_0 I_{enc} \tag{1}$$

Using stokes theorem we can get the differential form which is

$$\Delta \times \mathbf{B} - \mu_0 \mathbf{J} \tag{2}$$

or for free current density we will get

$$\Delta \times \mathbf{H} - \mathbf{J}_f \tag{3}$$

Where $\mathbf{J}_f$ is the free current. But there are two problem with this

1. taking divergence of the equation 2 we get

   $$\delta \cdot (\Delta \times \mathbf{B}) - 0$$

   So we get $\Delta . J = 0$. Which is not true and the correct relationship is

   $$\Delta \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

2. One more problem from above equation we get is when $\mathbf{J} = 0$ then we get $\Delta \times \mathbf{B} - 0$ which is also not correct. Because we know that in space

   $$\Delta \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

To solve this problem Maxwell introduced displacement current which is given by

$$\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E} = \epsilon \mathbf{E} \tag{4}$$

and displacement current density as

$$\mathbf{J}_D = \epsilon \frac{\partial \mathbf{E}}{\partial t} \rightarrow \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \text{ (in space)} \tag{5}$$

Then he included that term in the Amperes circuital law which then become as

$$\boxed{\oint \mathbf{B}.d\mathbf{l} = \mu_0 \iint \left(\mathbf{J} \cdot d\mathbf{l} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}\right) \cdot d\mathbf{S}} \tag{6}$$

And the differential equation becomes

$$\boxed{\Delta \times \mathbf{B} = \mu_0 \mathbf{J} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}} \tag{7}$$

Which solved the problem.