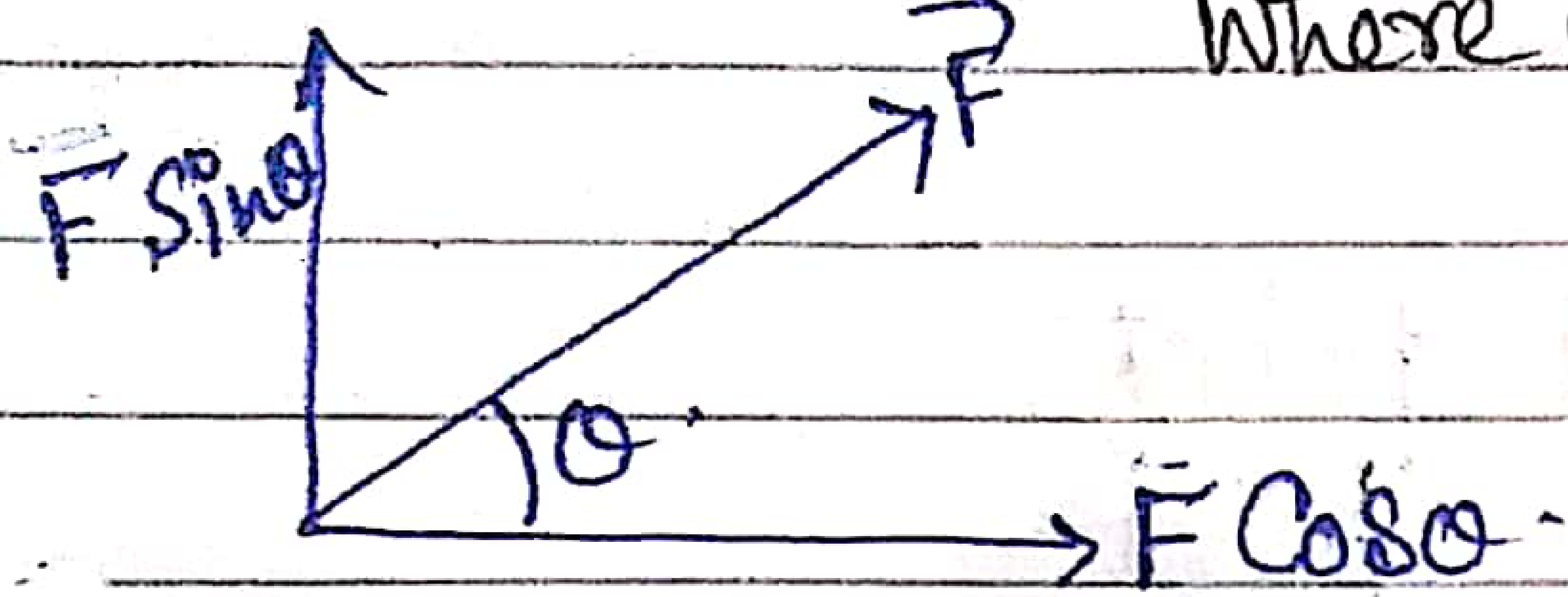
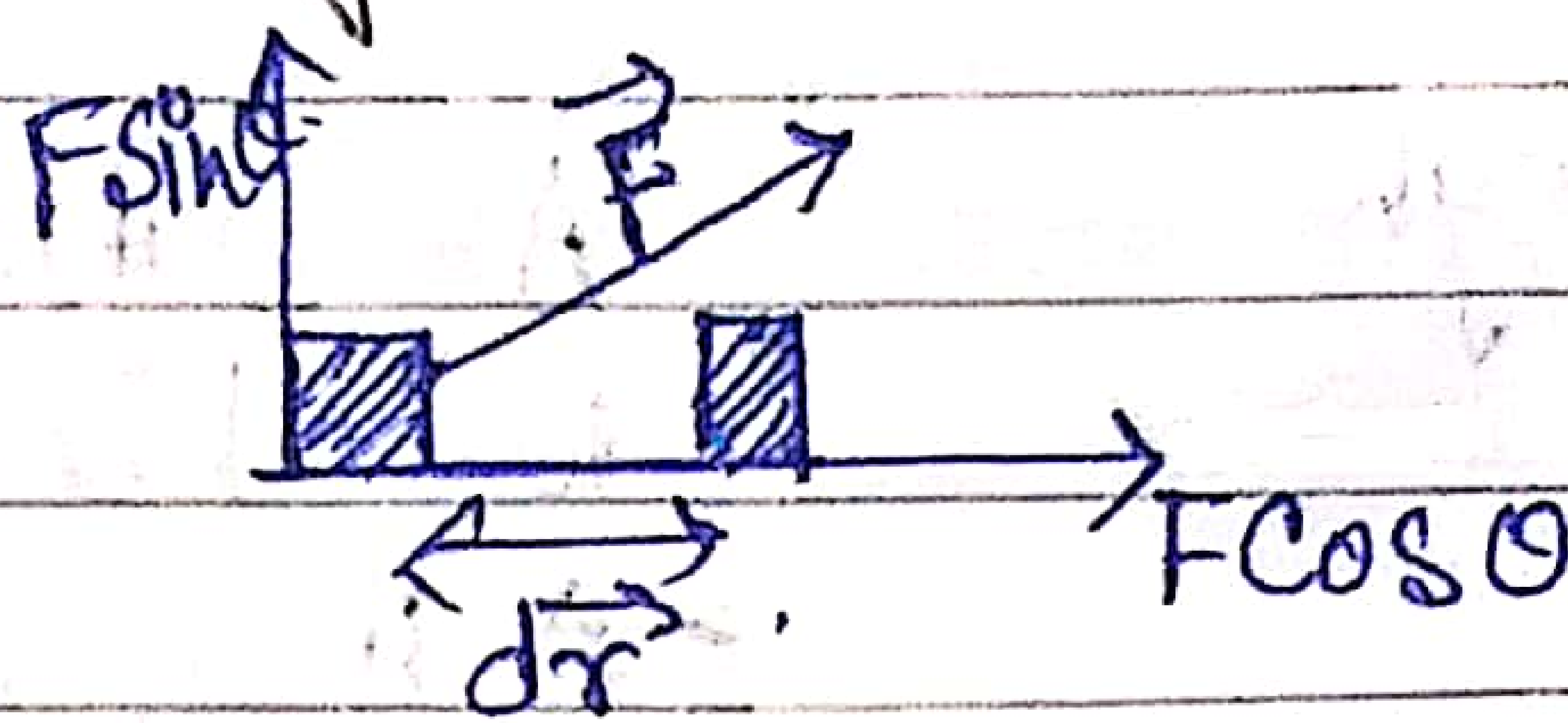
Work.

If $\vec{F}$ = applied force.
$\vec{dr}$ = displacement due to applicatⁿ of force.

then Work Done. $\delta W = \vec{F} \cdot \vec{dr}$.
$= F\,dr \cos\theta$, —(32)
where θ = angle between $\vec{F}$ & $\vec{dr}$.

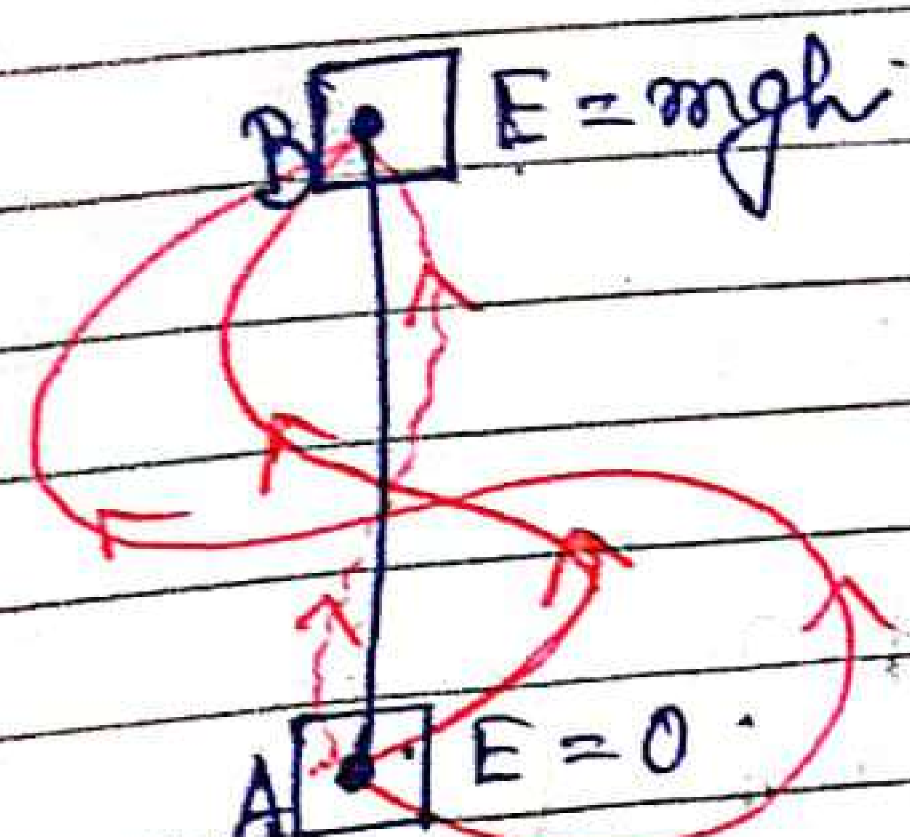




dr = magnitude of displacement.
$F\cos\theta$ = projection of $\vec{F}$ along the dirⁿ of displacement

CONSERVATIVE FORCE

A conservative force is a force if the
(1) work done by the forces in causing displacement of the object depends only on the initial and final position of the object
(2) and not on the path traversed from the initial to the final point.



A = Initial point
B = Final point.

→ Here, work done depends on the displacement from A to B
→ does not depend on the path taken by the particle.

- Examples of conservative and non-conservative force

Conservative	Non-conservative.
(1) Gravitational Force	(1) Frictional.
(2) Elastic Force (spring force)	(2) Air resistance.
(3) Electric Force	(3) Push or pull by a person.
(4) — Independent of the path	(4) — This force depends on the path traversed by the object
(5) Work done along any arbitrary closed path $\oint \vec{F} \cdot d\vec{r} = 0$.	(5) Work Done $\oint \vec{F} \cdot d\vec{r} \neq 0$.

Force as a gradiant of scalar potential

If $\vec{F}$ = conservative force.

$d\vec{r}$ = displacement due to the application of the force,

then in case of conservative force.

$$\oint \vec{F}\cdot d\vec{r} = 0 \quad \text{--- (32).}$$

In vector, Stoke's Theorem $\Rightarrow \oint_{l} \vec{A}\cdot d\vec{l} = \iint_{S} (\vec{\nabla}\times\vec{A})\cdot d\vec{s}$

Applying Stoke's Theorem

$$\oint \vec{F}\cdot d\vec{r} = \iint_{S} (\vec{\nabla}\times\vec{F})\cdot d\vec{s} = 0 .$$

$\because d\vec{s} \neq 0$, then $\vec{\nabla}\times\vec{F} = 0$.

or, $\vec{F} = -\vec{\nabla}V = -\text{grad } V$,

where V = potential energy.

POTENTIAL ENERGY

Let $\vec{F}$ = conservative force.

$d\vec{r}$ = displacement.

$V(\vec{r})$ = potential energy.

In a conservative field,

$\boxed{\text{potential energy} = \text{amount of work done}}$

$$\therefore \quad V(\vec{r}) = -\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} \quad \text{--- (33)}$$

$-$ve sign arises because as amount of work done increases, P.E. decreases.

$$V(\vec{r}) - \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

$$= \int (\hat{i} F_x + \hat{j} F_y + \hat{k} F_z) \cdot (\hat{i} dx + \hat{j} dy + \hat{k} dz)$$

putting $\vec{F} = \hat{i} F_x + \hat{j} F_y + \hat{k} F_z$.

$d\vec{r} = \hat{i} dx + \hat{j} dy + \hat{k} dz$.

$$\therefore V(\vec{r}) = - \int (F_x dx + F_y dy + F_z dz) \quad \text{--- (33)}$$

Now, $V(\vec{r}) = V(x, y, z)$.

$$\therefore dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz \quad \text{--- (34)}$$

Differentiating (33) $\Rightarrow$

$$dV = - (F_x dx + F_y dy + F_z dz) \quad \text{--- (35)}$$

Comparing (34) & (35), we get,

$$\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz = -(F_x dx + F_y dy + F_z dz).$$

$$= -F_x dx - F_y dy - F_z dz.$$

$$\therefore \frac{\partial V}{\partial x}dx = -F_x dx.$$

or, $\frac{\partial V}{\partial x} = -F_x.$

Similarly,

$$\frac{\partial V}{\partial y} = -F_y$$

$$\& \frac{\partial V}{\partial z} = -F_z$$

$$\therefore \vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z.$$

$$= \hat{i}\left(-\frac{\partial V}{\partial x}\right) + \hat{j}\left(-\frac{\partial V}{\partial y}\right) + \hat{k}\left(-\frac{\partial V}{\partial z}\right).$$

$$= -\left[\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right].$$

$$= -\vec{\nabla}V.$$

$$\therefore \boxed{\vec{F} = -\vec{\nabla}V}.$$