Wave motion

Dr. Soma Mandal,
Assistant professor,
Department of Physics, Government Girls' General Degree College, Kolkata

1 Wave

A wave is a disturbance within some media (like change of the pressure P in the air or displacement u of atoms in a solid) that propagates (or travels) with some velocity, which depends on the medium.

Waves can be periodic waves (like a steady sound wave) or solitary waves that are nonperiodic localized perturbations travelling through the media. In between these two types of waves are socalled wave packets that are pieces of periodic waves localized in space.

A wave is generated due to two simultaneous, at the same time, distinct motions. The first one is the oscillatory motion of the particles of the medium and the second is the linear motion of the disturbance. In wave motion, the propagation of a disturbance does not take place due to the physical movement of the particles in the medium. The disturbance actually propagates because of the transfer of energy from one particle to the other progressively. Thus, we may conclude that the waves transport energy and not the matter.

1.1 Plane and spherical waves

A plane wave is defined as a wave in which a wave amplitude is constant over all points of a plane perpendicular to the direction of propagation.

Spherical waves are waves in which the surfaces of common phase are spheres and the source of wave is a central point.

1.2 Longitudinal and Transverse Waves

The two basic types of waves are longitudinal and transverse waves.

1.2.1 Longitudinal Waves

In longitudinal waves, the oscillation of the particles is parallel to the direction in which the wave travels. Disturbance travelling in a spring parallel to its length, a pressure variation propagating in a liquid are examples of longitudinal waves. Longitudinal waves do not require shearing stress and hence can travel in any elastic medium – solid, liquid and gas.

Sound waves are also longitudinal waves. A loudspeaker supplied with alternating current creates sound waves because the diaphragm of the loudspeaker is forced to move to and fro. The diaphragm compresses the surrounding air in front of it as it moves forward and then it moves back before creating another compression. Effectively, the air which is the medium of propagation in this case, moves to and fro as the sound waves pass through it.

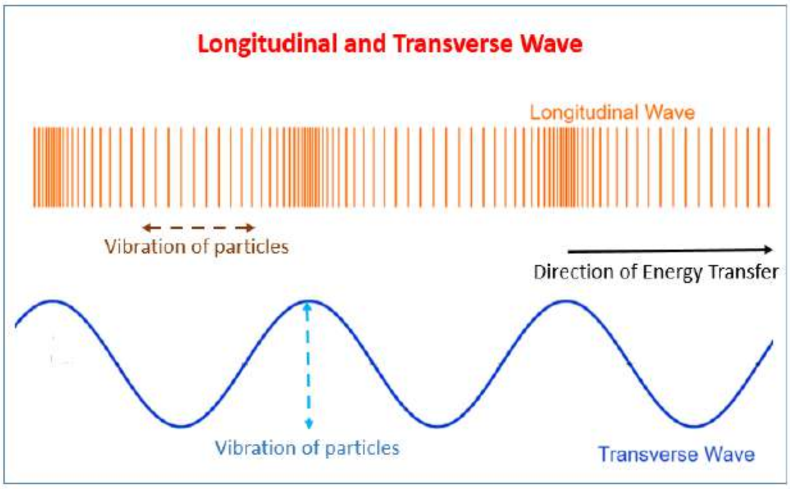

Figure 1: *Longitudinal and transverse waves*

1.2.2 Transverse Waves

In transverse waves, the particles of the medium oscillate perpendicular to the direction in which the wave travels. Travelling waves on a taut string, are transverse waves. When the one end of the string is rigidly fixed and the other end is given periodic up and down jerks, the disturbance propagates along the length of the rope but the particles oscillate up and down. The disturbance travels along the rope in the form of crests (upward peak) and troughs (valley).

Electromagnetic waves, which do not require any medium to propagate, are also an example of transverse waves. The electric and the magnetic field of an electromagnetic wave vibrate at right angles to the direction of propagation and also at right angles to each other.

1.2.3 More about longitudinal and transverse waves

In longitudinal waves, the displacement of the media occurs in the same direction in which the wave is propagating. The amplitude of this displacement is much smaller than the wavelength. The velocities of the media particles (that depend on time periodically, changing their sign) are much smaller than the speed of the wave (speed of sound). Longitudinal waves is the only kind of waves that exist in liquids and gases.

In transverse waves, the displacement of the media occurs in the direction perpendicular(or transverse) to the direction of the wave propagation. Again, the amplitude of this displacement is much smaller than the wave length, and the velocities of the media particles are much smaller than the speed of the wave (speed of sound).

Similarly to oscillations, waves require a restoring force. There is always an elastic restoring force for the longitudinal waves, since compressing the media leads to the increase of the pressure. Thus longitudinal waves can exist in all substances. For transverse waves, the elastic restoring force exists only in solids that retain their shape, apart from small deformations. In liquids and gases, there is no elastic restoring force for shifting layers with respect to each other. Thus there are no transverse elasic waves in the liquids and gases.

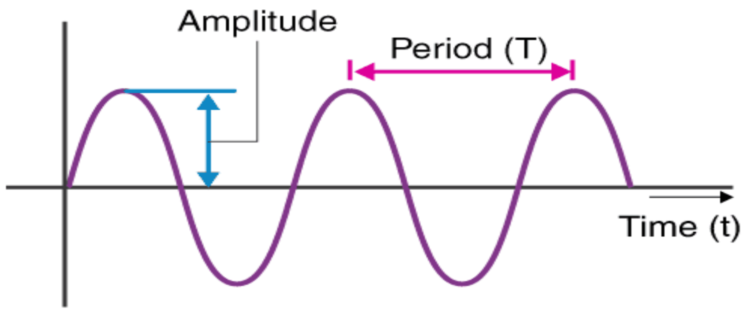

Figure 2: *Time-period*

2 Wave properties

When a wave moves, the displacements of the particles change with time as well as with the position. In one complete cycle of oscillation, the particles in the medium are displaced in one direction from their mean position to a position of maximum displacement, come back to the mean position and move in the opposite direction to the other extreme, and again come back to their mean position.

2.1 Wave speed

The speed of a wave is the distance it covers in one second. It should be carefully noted that the wave speed is completely different from the particle speed. Particle speed is the speed of the vibrating particles in the medium. On the other hand, wave speed is the speed with which the disturbance (or wave) propagates in the medium.

2.2 Wave frequency

The frequency with which the particles of the medium (through which the wave is passing) oscillate is known as wave frequency. In transverse waves, frequency is the number of crests (or troughs) that pass through a point in one second. In longitudinal waves, frequency is the number of compressions (or rarefactions) that pass through a point in one second. It is denoted by the symbol f. The SI unit of frequency is hertz (Hz), which is equal to 1 cycle per second. We already know that the wave motion requires a source which moves or vibrates with a particular frequency. So an important point to keep in mind is that the frequency of a wave is a property of the source, not of the medium through which it propagates

2.3 Time-period

The time period of the oscillation of the particles in the medium is the time period of the wave. It is denoted by the symbol T. The frequency of a wave is the reciprocal of the time period, i.e. $f = \frac{1}{T}$.

2.4 Amplitude

The amplitude of the wave is equal to the maximum positive displacement of the particles from their mean position. Thus, the amplitude of the wave is the same as the amplitude

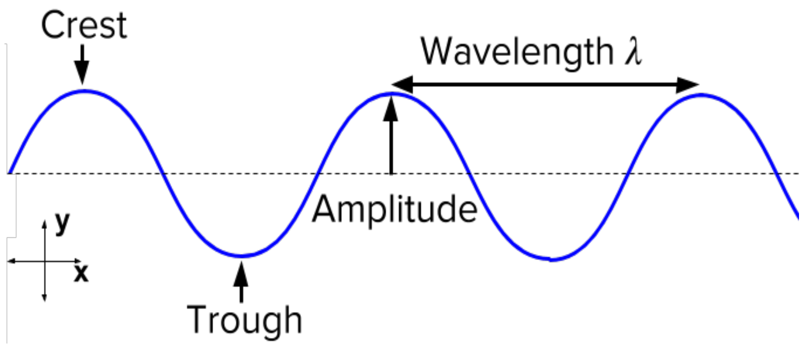

Figure 3: *Amplitude and wavelength*

of the oscillating particles.

2.5 Wavelength

The distance between any two points in the same state of motion defines the wavelength of a wave. Physically, this means that the wavelength is equal to the distance between two consecutive crests (or troughs). Wavelength is denoted by the symbol λ. The wave speed is given by

$$v = \frac{\lambda}{T}$$

Since, the frequency f of a wave is the reciprocal of its period T, the above equation can also be written as

$$v = f\lambda$$

The above equation predicts that in a given medium, the wave speed of a wave of given frequency is constant.

3 Mathematical description of wave motion

Let us consider a wave travelling along the positive $x-$ axis at the instant $t = 0$ and at time t. If the wave velocity is v, then as the wave travels the $y-$ coordinate of a point at $t = 0$ is the same as the $y-$ coordinate of other point at time t.

Therefore the wave equation can be written in a number of equivalent forms as

$$\begin{aligned} y(x,t) &= A\sin\left(\frac{2\pi(x - vt)}{\lambda}\right) \\ y(x,t) &= A\sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \\ y(x,t) &= A\sin(kx - \omega t) \end{aligned}$$

Differentiating twice w.r.t t and x and then combining we get

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \tag{1}$$

where $c = \frac{\omega}{k}$ is the velocity of the wave and the equation is the well known one dimensional differential wave equation.

Books Suggested:
(1). Principles of acoustics, Basudev Ghosh
(2). Sound, K. Bhattacharyya
(3). Waves and Oscillations, R. N. Chaudhuri