Propagation of longitudinal waves

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1 Propagation of longitudinal waves in a gaseous medium

We consider the propagation of longitudinal plane progressive sound waves through a gaseous medium. We consider a cylindrical section of the gas of uniform cross sectional area α. Let us choose an elementary slice of the cylinder of width δx, bounded by the planes AA' and BB'. The axis of the cylinder is chosen to be the $x-$ axis along which the sound waves are travelling.

Due to the excess force exerted by the sound waves, let the displacement of the layer AA' be ψ at any instant t. Since the displacement will be different for different layers, the layer BB' will undergo a displacement $(\psi + \delta\psi)$. With respect to any arbitrary origin, let the coordinates of A and B be respectively x and $(x + \delta x)$. Hence the coordinates of the points A_1 and B_1 are $(x + \psi)$ and $(x + \delta x + \psi + \delta\psi)$.

But $\delta\psi = \frac{\partial\psi}{\partial x}\delta x$.

Hence, the altered thickness of the section AB=$(\delta x + \frac{\partial\psi}{\partial x}\delta x)$.

$\therefore$ change in thickness=$\frac{\partial\psi}{\partial x}\delta x$.

$\therefore$ change in volume=$\alpha\frac{\partial\psi}{\partial x}\delta x$.

If δp be the excess pressure which causes this volume change, the excess force acting on the layer AA' will be $\delta p.\alpha$.

Now the volume strain=$\alpha\frac{\partial\psi}{\partial x}\delta x/\alpha\delta x$=$\frac{\partial\psi}{\partial x}$

Thus if K be the bulk modulus of elasticity of the medium

$$K = -\frac{\delta p}{\frac{\partial\psi}{\partial x}} \tag{1}$$

The $-ve$ sign arises because the changes of pressure and volume are of opposite sign. or

$$\delta p = -K\frac{\partial\psi}{\partial x}$$

This δp is the force per unit area acting on the layer AA' (now A_1A_2). Hence the total force on the layer is

$$\alpha\delta p = -K\alpha\frac{\partial\psi}{\partial x}$$

which acts along $-ve$ x-direction.

$\therefore$ force acting on the layer BB' (now B_1B_2) is

$$\begin{aligned} &+ \quad K\alpha\left[\frac{d}{dx}(\psi + \frac{\partial\psi}{\partial x}\delta x\right] \\ &= \quad +K\alpha\frac{\partial\psi}{\partial x} + K\alpha\frac{\partial^2\psi}{\partial x^2}\delta x \end{aligned}$$

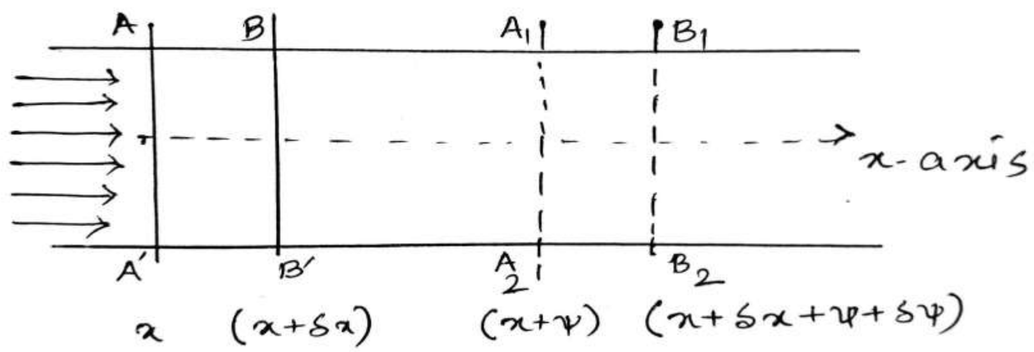

Figure 1: *Longitudinal waves in a gaseous medium*

This force acts along $+ve$ x-direction.

$\therefore$ net force acting on the slice AB along $+ve$ x direction

$$\begin{aligned} &= K\alpha\left(\frac{\partial\psi}{\partial x}+\frac{\partial^2\psi}{\partial x^2}\delta x\right)-K\alpha\frac{\partial\psi}{\partial x} \\ &= K\alpha\frac{\partial^2\psi}{\partial x^2}\delta x \end{aligned}$$

If $\frac{\partial^2\psi}{\partial t^2}$ represents the acceleration of the layer chosen at time t, we may write

$$\rho\alpha\delta x\frac{\partial^2\psi}{\partial t^2}=K\alpha\frac{\partial^2\psi}{\partial x^2}\delta x$$

or

$$\frac{d^2\psi}{dt^2}=\frac{K}{\rho}\frac{d^2\psi}{dx^2} \tag{2}$$

where ρ is the density of the concerned gas.

Comparing it with the equation of motion of a one dimensional progressive wave

$$\frac{d^2\psi}{dt^2}=v^2\frac{d^2\psi}{dx^2},$$

we find that the velocity of the longitudinal waves through the gaseous medium is

$$v=\sqrt{\frac{K}{\rho}} \tag{3}$$

$\therefore$ equation (2) may be rewritten as

$$\frac{d^2\psi}{dt^2}=v^2\frac{d^2\psi}{dx^2} \tag{4}$$

The general solution of equation (4) is given by

$$\psi(x,t)=f_1(vt-x)+f_2(vt+x) \tag{5}$$

$f_1(vt-x)$ represents a wave travelling along $+ve$ x- direction while $f_2(vt+x)$ denotes another wave.

1.1 Acoustic Pressure

We know that the excess pressure over a gaseous layer at a position $x = x$ is $\delta p = -K\frac{\partial \psi}{\partial x}$. Sinece the equation of a plane progressive sound wave is given by

$$\begin{aligned} \psi &= a\sin\frac{2\pi}{\lambda}(vt-x) \\ \frac{\partial \psi}{\partial x} &= -a\frac{2\pi}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x) \\ \therefore \delta p &= \frac{2\pi a K}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x) \end{aligned} \tag{6}$$

$$\therefore \delta p_{max} = \frac{2\pi a K}{\lambda}$$

The mean squarred acoustic pressure variation is given by

$$\begin{aligned} (\bar{\delta p})^2 &= \frac{1}{T}\int_0^T \left(\frac{2\pi a K}{\lambda}\right)^2 \cos^2\frac{2\pi}{\lambda}(vt-x)dt \\ &= \frac{1}{T}\left(\frac{2\pi a K}{\lambda}\right)^2 \frac{T}{2} \\ &= 2\left(\frac{\pi a K}{\lambda}\right)^2 \end{aligned}$$

$\therefore$ acoustic pressure $= \sqrt(2)\frac{\pi a K}{\lambda}$.

Books Suggested:

(1). Principles of acoustics, Basudev Ghosh
(2). Sound, K. Bhattacharyya
(3). Waves and Oscillations, R. N. Chaudhuri