Waves
Practice Question Set I
Full marks 50
Paper:PHS-A-CC-2-4-TH

The figures in the margin indicate full marks.
Candidates are required to give their answer in their own words as far as possible.

1. Answer all the **five** questions. $5 \times 2{=}10$

(a) The displacement of a particle of mass m executing underdamped SHM is given by $x(t) = Ae^{-bt}\cos(\omega' t - \delta$ where $\omega' = \sqrt{{\omega_0}^2 - b^2}$, ω_0 is the natural frequency and b is the damping factor. Calculate A and δ subject to the initial condition $x = 0$, $\dot{x} = 0$ at $t = 0$. 2

(b) A $20g$ particle moves in SHM with a frequency of 3 Hz and an amplitude of 5 cm. What are its maximum speed and maximum acceleration? 2

(c) Distinguish between amplitude resonance and velocity resonance. 2

(d) The equation of a plane sound wave is $S = 6.0 \times 10^{-6}\cos(1900t + 5.72x)m$. Find the velocity of the wave. 2

(e) What are beats? 2

2. Answer all **four** questions. $4 \times 10{=}40$

1.(a) Write down the equation of motion of a one-dimensional simple harmonic oscillator under the effect of damping force proportional to its velocity. Solve the equation for the case of under-damped oscillation. Show that the time period of oscillation is greater when damping present.

(b) Show that in case of forced vibration $\frac{average\ K.E.}{average\ P.E} = \frac{\omega^2}{{\omega_0}^2}$, wher ω_0 is the natural frequency of oscillation and ω is the frequency of the driving force. 2+3+2+3

2. (a) A particle is subjected to two SHMs represented by $x = A\sin\omega t$ and $y = B\cos 2\omega t$ in a plane. Show that the resultant locus in the $X - Y$ plane is given by $x^2 = \frac{A^2}{2B}(B - y)$.

(b) Explain sharpness of resonance. What do you mean by bandwidth and quality factor?

(c) Establish the relation between group velocity and phase velocity. 3+2+2+1+2

3. (a) A plucked string of length L is excited at $\frac{L}{3}$ and touched at $x = \frac{L}{4}$. Calculate the harmonics present.

(b) The dispersion relation for transverse waves propagating in a medium is given by $\omega^2 = {\omega_p}^2 + k^2c^2$ where the symbols have their usual meanings. Show that $v_g v_p = c^2$.

(c) For a stretched string of length L fixed rigidly at two ends, the displacement at a point x at time t is

$$y(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\left[a_n\cos\frac{n\pi ct}{L} + b_n\sin\frac{n\pi ct}{L}\right]$$

in usual notation. Obtain the fundamental frequency in terms of tension of the string and mass per unit length. State the initial conditions that one should use the cases of a plucked string and a struck string. 4++1+1+1

4.(a) A mass of 1 kg is acted on by a restoring force with force constant 4 N/m and a restoring force with damping coefficient 2 $N-s/m$. Write down the equation of motion in one dimension, Find (i) whether the motion is periodis or oscillatory. (ii) the value of the resisting force which will make the motion critically damped.

(b) A particle is subjected to two simple harmonis motions at right angles to each other, having the same frequency. Show that the resultant locus of the particle is an ellipse. Hence find the locus when the two motions are (i) in phase and (ii) in opposite phase. $1+2+2+4+1$