Semester IV
Paper: PHS-A-CC-4-9-TH
Element of Modern Physics: Nuclear structure

(a) Size and structure of atomic nucleus and its relation with atomic weight; Impossibility of an electron being in the nucleus as a consequence of the uncertainty principle.
(b) Nature of nuclear force, NZ graph.
(c) Nuclear Models: Liquid Drop model. semi-empirical mass formula and binding energy. Nuclear Shell Model. Magic numbers.

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1 Introduction: General properties of Nuclei

There are only three fundamental atomic particle electron, proton and neutron. After the discovery of the neutrons by Chadwick in 1932, it was proposed that nuclei are composed of protons and neutrons, which are collectively called nucleons. The neutron carries mass slightly greater than that of the proton, but unlike proton which is positively charged, a neutron is electrically neutral. A species of atom characterized by the constituents of its nucleus is called a nuclide.

The important static properties of the nuclei include their electric charge, mass, binding energy , size , shape, angular momentum, magnetic dipole moment, electric quadruploe moment, statistics, parity and iso-spin.

1.1 Nuclear Mass and binding energy

The atomic nuclei are made up of two different types elementary particles, protons and neutrons. The sum of the number of neutrons (N) and protons (Z) inside the nucleus is known as its mass number A, so that $A = N + Z$. Z is equal to the atomic number of the element in the periodic table. A nucleus of an atom X of atomic number Z and mass number A is written as ${}^A_Z X$.

The nuclear mass M_{nuc} is obtained from the atomic mass M by subtracting the masses of the Z orbital electrons from the atomic mass. $M_{nuc} = M - Zm_e$. The nuclei of atoms are very strongly bound. If we want to break up a nucleus of Z protons and N neutrons completely so that they are all separated from one another, a certain minimum amount of energy is to be supplied to the nucleus. This energy is known as the binding energy of the nucleus.

It has been observed that the mass of a nucleus is always less than the sum of the individual masses of the protons and the neutrons, which constitute it. This difference is a measure of the nuclear binding energy which holds the nucleus together, is known as the mass defect Δm.

Hence the nuclear binding energy is given by $E_B = \Delta mc^2$, where c is the speed of light. Let M_p be the rest mass of a proton, M_n is that of proton and M_N is the rest mass of the nucleus; Z and A are the atomic and mass numbers of the nucleus.

$$E_B = \left[ZM_p + (A - Z)M_n - M_n\right] c^2 \tag{1}$$

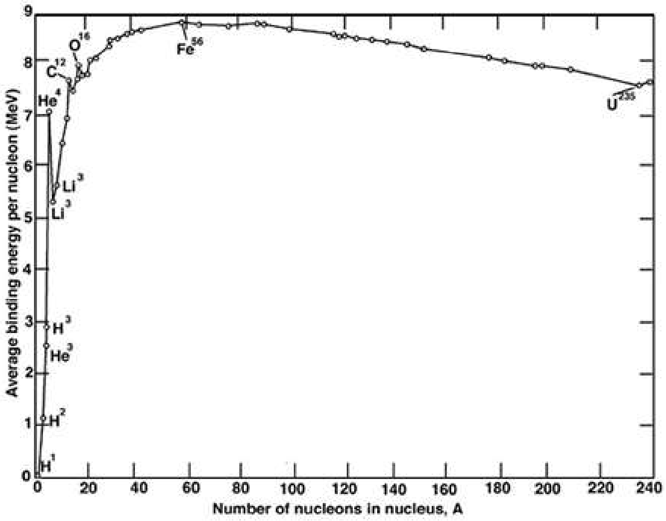

Figure 1: *N versus Z plot for stable nuclei*

Thus the average binding energy per nucleon is

$$\epsilon = E_B/A \tag{2}$$

The graphical plot of ϵ against the mass number A is depicted in Figure (1).

As evident from the binding energy curve most of the nuclei have an average binding energy per nucleon of $7.5 - 8.9 Mev$. The elements with low A values have their ϵ values less than 7 Mev, the important exception being He^4, C^{12} etc nuclei which have their A values integral values of 4. Again ϵ is maximum for those elements which have $A \sim 50$ (for those elements which be in the middle of the periodic table) ϵ assumes a value $8Mev$. For elements with high atomic and mass number mainly for radioactive isotopes ϵ is less than $8Mev$ which implies that these elements are unstable.

2 Size

Nuclear size is defined by nuclear radius, also called rms (root mean square) charge radius. It can be measured by the scattering of electrons by the nucleus. The problem of defining a radius for an atomic nucleus is similar to the problem of atomic radius, since neither atoms nor their nuclei have well-defined boundaries. Generally, the nucleus is modeled as a sphere of positive charge for the interpretation of electron scattering experiments.

Rutherford's theory of α particle sacttering gives us an idea about the smallness of the nuclear size. In later years more accurate method for the measurement of the nuclear radius have been developed. It has been assumed that the nucleus has a spherical shape. The small depurture from the sphericity of certain nuclei is inferred from the existence of electric quadrupole moment of these nuclei which is zero for the spherical nuclei. It has also been assumed that the nuclear charge is uniformly distributed. Experiments show that this is very nearly so and the nuclear charge density ρ_c is approximately constant. Experimental evidences also show that the distribution of nuclear matter density ρ_m is also approximately constant. Since nuclear mass is almost linearly proportinal to mass number A, this means that $\rho_m \sim A/V = constant$, i.e., the nuclear volume $V \propto A$. assume a spherical shape of the nucleus with a radius R, we then get

$$V = \frac{4}{3}\pi R^3 \propto A$$

$$R \propto A^{1/3}$$

so that

$$R = r_0 A^{1/3} \tag{3}$$

where r_0 is a constant, known as the nuclear radius parameter.

This nuclear radius is the radius of the nuclear mass distribution.

3 Nuclear Structure

Basic ideas about the structure of atomic nuclei:

Rutherford's experiment on the scattering of α particles revealed the existance of a very small entity having a very small $+ve$ charge on it within the atom. This small entity carrying a positive charge was called the nucleus. Calculation from experimental results showed that the approach of an α particle to the nucleus of an atom was of the order of 10^{-12} cm whereas from the study of kinetic theory of gases and other phenomena the diameter of atom was shown to be of the order of 10^{-8} cm. This proved conclusively that an atom consists of a small massive nucleus which carries a positive charge with a comparatively vast region around it containing the electrons. Natural radio-activity in this turn, clearly points out that the nuclear structure though compact must be very complex, capable of ejecting different particles like α particles and electron as well as electromagnetic radiation in the form of γ rays. Several theories have been put forward for the explanation of nuclear structure which are discussed below:

3.1 The proton-electron theory

The fact that certain radioactive atoms emit α and β particles both of which are corpuscular in nature, led to the idea that atoms are build up elementary constituents. Prout suggested that all atomic weights are whole numbers, that they might be build up of hydrogen. Prout's hypothesis was discarted when it was found that that the atomic weights of some elements are fractional. The discovery of isotopes suggests that some of the ordinary elements consists of a mixture of isotopes. According to Aston's whole number rule all atomic weights determined by chemical methods are caused by the presence of two or more isotopes each of which has a nearly integral atomic weight.

To account the mass of a nucleus whose atomic weight is very close to integer A, called the mass no, it was necessary to assume that the nucleus contained protons. But if this was the case, the charge on the nucleus would be equal to A and not equal to the atomic no. Z. To overcome this difficulty, it was assumed that in addition to the proton, atom contained $(A - Z)$ electrons. These electrons would contribute a negligible amount to the mass of the nucleus but would make the charge equal to $+Z$ as required. It was thus possible to consider the atoms as consisting of a nucleus of A protons and $(A-Z)$ electons surrounded by Z extra nuclear electrons. The β ray emission by the natural radioative elements seem to confirm the existence of electrons in the nucleus.

This proton electron hypothesis of the nuclear constitution has however many flaws. Electrons cannot remain within the nuclei which have radii of the order of 10^{-14} or less. According to Hysenberg's uncertainty principle, the uncertainty in the momentum p of the electron in the nucleus would then be

$$\Delta P = \frac{\hbar}{\Delta x} = \frac{\hbar}{R} \sim \frac{10^{-34}}{10^{-14}} = 10^{-20} kg.m/s \tag{4}$$

An electron with momentum of this order of magnitude would have the energy

$$E \sim c\Delta P = \frac{3 \times 10^{8} \times 10^{-20}}{1.6 \times 10^{-13}} = 20MeV \tag{5}$$

There is no experimental evidence of the existence of such high energy electrons within the nuclei of atoms.

Again, from consideration of angular momentum of the nuclei, the proton-electron hypothesis would pose serious difficuties. Electron and proton are both spin 1/2 particles. Their total number in the nucleus should be $A + A - Z = 2A - Z$. If this is even, then the total spin(I) of the nucleus should be integral, while if it is odd, I should be half integral. But the measurements gives the result which is contrary to the expectation from the proton-electron hypothesis.

3.2 Proton-neutron theory

The theory discussed earlier failed to solve the difficulties connected with nuclear spin, magnetic moments, statics etc. Then with the discovery of neutron, the set up of the nucleus got changed. Now it is established that nuclei are composed of protons and neutrons. In general, if an atom has an atomic weight A and atomic no. Z, the nucleus contains Z protons and $(A - Z)$ neutrons. The ejection of a electron as a β particle from the nucleus of an atom is due to the internal conversion of a neutron into a proton as postulated by Pauli in course of his explanation of β ray spectrum.

It is possible to explain the observed spin, magnetic moment and statistics of the nuclei on the basis of proton-neutron hypothesis. Since there are A number of protons and neutrons in the nucleus, each of intrinsic spin 1/2(in unit of $\hbar$), the total nuclear spin I should be 0 or integral if A is even and half-integral if A is odd, the orbital angular momentum of the nucleons being always integral.

The magnetic moments of the proton and neutron are of same order of magnitude.

$$\mu_p = 2.7927\mu_N$$

$$\mu_n = -1.9131\mu_N$$

where $\mu_N = e\hbar/2M_P$ is the nuclear magneton, M_p being the protonic mass. So a nucleus made up of protons and neutrons only should have a magnetic moment of the same order of magnitude as μ_N. This is corroborated by experimental measurements.

Finally, the statistics obeyed by the nuclei can be explained on the basis of proton-neutron hypothesis. Since the nucleons are fermions, a nucleus with even A should obey B_E statistics, while a nucleus with odd A should obey Fermi-Dirac statistics. This is in agreement with observations.

The proton-neutron hypothesis can easily explain the existence of more than one isotope of an element. The chemical nature of an element is determined by the number of proton within the nucleus. If the proton number Z remains the same, but the neutron number $N = A - Z$ becomes different, the chemical nature of the atoms remains the same, but the atomic masses become different. Such nuclides are known as isotopes.

3.3 Neutron-Positron hypothesis

According to Prof. Perrin, an atom with mass no. Z shouls contain A neutrons and Z protons (positive electrons) in its nucleus. In this scheme also the neutrality of the atom as a whole is secured. But this theory was not accepted owing to some difficulties arising out of it as in the concept of the proton-electron theory.

4 Nuclear Force

There are three types of nuclear force
(1). Force between two protons (p-p force)
(2). Force between a proton and neutron (p-n force)
(3). Force between the neutrons (n-n force).

4.1 Nature of Nuclear force

The protons and neutrons are very strongly bound within the nucleus. The nature of the force which binds them together is basically different from gravitational or electromagnetic forces. The gravitational force is far too weak to account for the nuclear binding.

So far as the electromagnetic force is concerned, two protons repel one another due to like charges on them. again the neutrons being electrically neutral cannot have any electromagnetic interaction between themselves or with the protons.

Thus a new type of force different from gravitational or electromagnetic force act between the nucleons within the nucleus.This force is very strongly attractive upto a certain maximum distance between the nucleons which of the order of about 2 fm. This distance is known as the range of the force. Beyond this distance the force is negligibly small. It is known as strong interaction. This force is attractive between any two of the nucleons, whether they are two protons, two neutrons or one proton and one neutron.

Some of the characteristics of theses nuclear forces are as follows:

(a). Nuclear forces are charge independent: The force between two neutrons is the same as between the two protons and between a proton and a neutron , i.e. $(p, p) \equiv (n, n) \equiv (p, n)$.

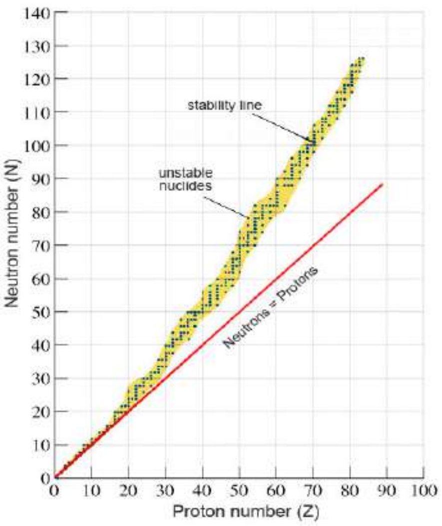

Figure 2: *N versus Z plot for stable nuclei*

(b). Nuclear forces are short range: The tiny size of the nucleus (of the order of fm) and its great stability shows that the forces operating among the nucleons inside the nucleus are short-range forces, i.e. they are effective over a very short distances only (a few fm). This short-range character of the nuclear forces distinguishes it from gravitational and electromagnetic forces, which act over large distances.

(c). Nuclear force are exchange forces: Yukawa in 1935 proposed that this force acts through the interchange of particle of mass intermediate between that of an electron and a proton, just like the electromagnetic interaction acts through the intermediary of a virtual photon.

(d). Nuclear forces are very strong: Nuclear forces are much stronger than the electromagnetic and gravitational forces. They are stronger by a factor of 137 from the electromagnetic force and are stronger by a factor of $10^{4}0$ from gravitational forces.

5 Stable Nuclei:NZ graph

Out of all nuclides, only about 25% are stable.

A stable atom is an atom that has enough binding energy to hold the nucleus together permanently. Many nuclei in nature are very stable, most of the nuclei formed at the creation of the universe or after supernovae explosions many millions of years ago are still in existence now.

Fig (2) shows the plot of the number of neutrons ($N = A - Z$) against the number of protons Z for the stable nuclei of different elements. The ratio N/Z for the stable nuclides is confined within a narrow range about the mean solid line drawn through the points (stability line). For the lighter nuclei the number of protons and neutrons are nearly equal so that $N/Z = 1$ for them and the stability line is equally inclined to the z and N axes. For the heavier nuclei, the number of neutrons is somewhat higher than the number of protons so that N/Z becomes greater than 1 for higher Z. Its highest value is about 1.6

for very heavy nuclei. The stability line is thus steeper at higher Z.

This figure also shows that the isotopes of different elements (Z=constant) lie on different vertical lines. On the other hand, the nuclides with different Z having the same mass number (A=constant) lie along lines inclined at 135 deg to the Z axis. These are known as isobars. Finally, the nuclides with the same number of neutrons (N=constant) lie along the different horizontal lines. They are known as isotones. The plot of the nuclei in the N vs. Z graph is known as Segree chart.

The number of stable isobars for different A are usually one or two. In a few cases three stable isobars are found.

The isotopes of all elements can be divided into four groups: even$Z$-even N (e-e), even z -odd N (e-o), odd Z-even N(o-e) and odd Z-odd N (o-o).

The equality of Z and N for the lighter nuclei shows that the prton-proton and neutron-neutron forces are approximately equal within the nuclei. This is known as charge symmetry of the nuclear force. In the heavier nuclei the Coulomb repulsion between the protons tend to weaken the binding. In this case the number of neutrons must be relatively higher, which increases the strength of binding. Stabilty of a nucleus is achieved when a definite number of protons and a definite number of neutrons are present in it. When their relative proportion is disturbed too much, the stability of the nucleus is also affected.

If the number of neutrons N is increased, keeping the number of protons constant, then the nucleus thus formed goes to the left of the stability line. In such a nucleus, a neutron may spontaneously transform into a proton and the nucleus becomes β^- active. On the other hand , if the number of protons is increased, keeping N constant, the new nucleus thus formed falls on the right of the stability line. In such a nucleus, a proton may transform spontaneously into a neutron and the nucleus become β^+ active.

In unstable nuclei the strong nuclear forces do not generate enough binding energy to hold the nucleus together permanently. It is unstable nuclei that are radioactive and are referred to as radioactive nuclei and in the case of their isotopes called radioisotopes.

Books Suggested:
(1). Atomic and Nuclear Physics, Vol II; S. N. Ghosal
(2). Nuclear Physics, Theory and Experiment; R. R. Roy & B. P. Nigam
(3). Atomic and Nuclear Physics, An Introduction; T.A. Littlefield & N. Thorley
Image courtesy: Wikipedia