Semester IV
Paper: PHS-A-CC-4-9-TH
Element of Modern Physics: Nuclear Models

(a) Size and structure of atomic nucleus and its relation with atomic weight; Impossibility of an electron being in the nucleus as a consequence of the uncertainty principle.
(b) Nature of nuclear force, NZ graph.
(c) Nuclear Models: Liquid Drop model. semi-empirical mass formula and binding energy. Nuclear Shell Model. Magic numbers.

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1 Introduction

In order to understand the observed properties of the nucleus of an atom it is necessary to have an adequate knowledge about the nature of the internucleon interaction. But none of the proposed theories gives us a full understanding of the nature of the internucleon interaction. Even if the the nature of internucleon interaction were known, it would have been extremely difficult to develop a satisfactory theory of the structure of the nucleus made up of a large number of protons and neutrons, since it is almost impossible to solve the Schrödinger equation exactly for such a many body system. Even if one uses nucleons as a fundamental degrees of freedom, the system is too small for statistical approach too. Thus the problem is complicated and this is probably one of the main explanation for the existence of large number of theoretical approaches that have been (and are) used. Therefore we make use of models and use simple analogies.

1.1 Liquid drop model

The liquid drop model first proposed by Bohr, compares the nucleus to a liquid drop the nucleus corresponding to the molecules of the liquid, due to the several points of similarity such as large interaction between constituent particles, nearly constant density, surface tension effect etc. It is believed that each individual molecule within a liquid drop exerts an attractive force upon a group of molecules in its immediate neighbourhood. The force of interaction does not extend to all the molecules within the liquid drop. This is known as the saturation of the force. In order to calculate the potential of this interaction, it is necessary to know the number of interacting pair of molecules within the drop. If each molecule interacts with all the molecules in the drop, the number of interacting pair should be $N(N-1)/2$ where N is the total number of layer molecules. For N large, the number of pairs would be $N^2/2$ so that the potential energy should be proportinal to N^2. On the otherhand if each molecule interacts with a limited number of molecules in its immediate vicinity, the number of interacting pairs would be linearly proportional to N so that the interaction potential should be proportional to N. This latter conclusion is supported by experimental evidence.

We have seen that the binding energy E_B of a nucleus is proportional linearly to the number of nucleus within it, so that the binding fraction (i.e. the binding energy per nucleon) is nearly constant ($\sim 8Mev$) for most nuclei. This fact shows a close resemblance of the nucleus with a liquid drop. Therefore the internucleon force within the nucleus attains a saturation value, so that each nucleon can interact only with a limited number of nucleons in its close vicinity. Moreover there are following point of resemblance between the nucleus of an atom and a liquid drop:

(i). The attractive force near the nuclear surface is similar to the force of surface tension of the surface of the liquid drop.

(ii). As in the case of a liquid drop, the density of the nuclear matter is independent of its volume. The radius $R \propto A^{1/3}$ where A is the mass number. Hence the nuclear volume $V \propto A$. Since the nuclear mass $M \sim A$, the density of the nuclear matter $\rho_m = M/V$ is independent of A. This also suggests saturation of nuclear force.

(iii). Different type of particles, e.g., neutrons, protons, deuterons, $\alpha-$ particles etc. are emitted during nuclear reactions. These processes are analogous to emission of the molecules from the liquid drop during evaporation.

(iv). The internal energy of the nucleus is analogous to the heat energy within the liquid drop.

(v). The formation of a short lived compound nucleus by the absorption of a nuclear particle in a nucleus during a nuclear reaction is analogous to the process of condensation from the vapour to the liquid phase in the case of the liquid drop.

The liquid drop model is not very sucessful in describing the low lying excited states of the nucleus.

1.2 Bethe-Weisa$\ddot{a}$cker formula:

This semiemperical formula for the nuclear mass gives a connection between the nuclear matter with experimental information and is based on the liquid drop model of the nucleus.

The mass of a nucleus having Z protons and N neutrons is given by

$$M = ZM_H + NM_n - E_B \tag{1}$$

where $M_H \rightarrow$ mass of hydrogen ion
$M_n \rightarrow$ mass of a neutron
$E_B \rightarrow$ binding energy, defined as the minimum work nesessary to dissociate a nucleus into its component nucleons.

E_B is made up of a number of terms as discussed below, where we have assumed the nucleus to behave like a liquid drop of an incompressible liquid of constant density.

1. Volume Energy Term:-
The volume energy term due to exchange forces gives the largest contribution to E_B. The term can be identified as the first term in binding energy and may be expressed as

$$E_0 = a_v A \tag{2}$$

, where a_v is a constant which can be determined from the known values of masses; E_0 is called volume energy, since just like nuclear volume, this term is proportional to mass no A.

2. Surface Energy Term:-
The nucleus of an atom has some nucleons on its surface. These nucleons do not have as many neighbours as the nucleons in the interior. The exchange forces in the interior will get saturated but those on the surface will remain unsaturated. The volume energy E_0 will have a correction term E_1 representing the nucleoons on the surface.

Now, the radius of the nucleus is given by $R = r_0 A^{1/3}$, where r_0 is a constant. Therefore nuclear surface area $= 4\pi R^2 = 4\pi {r_0}^2 A^{2/3}$. Hence the number of nucleons exposed to the surface will be proportional to $A^{2/3}$. Thus the negative correction term will be given by

$$E_1 = -a_s A^{2/3} \tag{3}$$

, where a_s is a constant.

3. Coulomb energy term:
Coulomb repulsive forces are produced due to the mutual repulsion of positively charged protons in the nucleus. The protonic charge Ze is evenly distributed in the entire volume of a nucleus. Just like the surface energy, the coulombian energy E_2 will have a $-ve$ contribution and is given by

$$\begin{aligned} E_2 &= -\frac{3}{5}\frac{Z^2e^2}{R} \\ &= -\frac{3}{5}\left(\frac{Z^2e^2}{r_0 A^{1/3}}\right) \\ &= -a_c \frac{Z^2}{A^{1/3}} \end{aligned} \tag{4}$$

where

$$a_c = \frac{3}{5}\frac{e^2}{r_0} = constant$$

4. Asymmetric energy term:
In heavy nuclei the number of neutrons exceeds that of protons by a large factor. Only in a few light nuclei $Z = N$. It is essential to add more neutrons to provide stability to the nuclei against the Coulomb's repulsion due to a large number of protons in a heavier nuclei. The energy states of the individual nucleons in the nucleus are however quantised and the nucleons are arranged according to Pauli's exclusion principle. If we put Z protons in certain low energy states, an equal no. of neutrons can be accommodated in theses states along with the protons. The $(N - Z)$ excess neutrons will therefore go to the higher unoccupied quantum states which are the states of larger kinetic energy. Since the binding energy is equal to $(P.E - K.E)$, these $(N - Z)$ excess neutrons will have a much smaller binding. Thus, these $(N - Z)$ excess neutrons will produce a deficit in binding energy of a fraction of the nuclear volume.

The fraction of nuclear volume affected per nucleon is $\left(\frac{N-Z}{A}\right)$. Therefore total deficit in binding energy

$$\begin{aligned} \propto \left(\frac{N-Z}{A}\right)(N-Z) &= -a_a \frac{(N-Z)^2}{A} \\ &= -a_a \frac{(A-2Z)^2}{A} \end{aligned}$$

, where a_a is a constant. Hence the assymmetric energy is

$$E_3 = -a_a \frac{(A-2Z)^2}{A} \tag{5}$$

5. Pairing energy :

The nuclear binding energy is somewhat affected by the spin angular momentum of the nucleons. For nuclei of even A, Z and N may be both even or both odd. If these numbers are even, the nucleus may be grouped into stable pair, with sign opposed and the nucleus will be correspondingly more stable than in the case of Z and N being odd. The pairing energy contribution is zero for odd A. The pairing energy is actually represented by
$E_\delta = +33A^{-3/4}$ Mev for even (Z)-even (N) nuclei;
$= 0$ for even-odd/odd-even nuclei;
$= -33A^{-3/4}$ Mev for odd-odd nuclei.

Hence,

$$E_B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + E_\delta \tag{6}$$

Thus, the Bethe-Weisa$\ddot{a}$cker semi emperical mass formula becomes,

$$\begin{aligned} M(Z,A) &= ZM_H + NM_n - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{A^2 - 4AZ + 4Z^2}{A} - E_\delta \\ &= \left[(M_n - a_v + \frac{a_s}{A^{1/3}} + a_a)A - E_\delta \right] + \left[M_H - M_n - 4a_a \right] Z + \left(\frac{a_c}{A^{1/3}} + \frac{4a_a}{A} \right) Z^2 \\ &= \alpha A + \beta Z + \gamma Z^2 - E_\delta \end{aligned} \tag{7}$$

where

$$\alpha = M_n - a_v + \frac{a_s}{A^{1/3}} + a_a$$
$$\beta = M_H - M_n - 4a_a$$
$$\gamma = \frac{a_c}{A^{1/3}} + \frac{4a_a}{A}$$

Thus the relation between $M(Z,A)$ and Z is a parabolic one.

1.3 Nuclear Shell structure

The most useful model concerning the nuclear structure is the shell model of the nucleus. This model is preferred because of the reason that the nucleus actually exists in groups or shells within the nucleus.

The nuclear shell model compares with the electron shell model of the atom in that shells are regarded as filled when they contain a specific number of nucleons. A nucleus which has filled shells is more stable than one which has unfilled shells. Extra stable

nuclei are thus analogous with the inert gas atoms, which have filled electron shells. The elucidation of the laws governing the number of nucleons in filled nuclear shells is, however, not so readily possible as it is in the case of electrons. Though the Pauli's exclusion principle applies, the theoretical task is much more difficult because
(i). there are two different type of nucleons concerned, the proton and neutron, as compared with electrons only in the extra-nuclear structure of the atom.
(ii). Unlike the extra-nuclear electrons, the nucleons are not subject to the attraction of central force of electrostatic origin, indeed the nature of the internucleon force is not fully understood.

Evidence for the number of nucleons in closed shell is consequently semi-emperical and based on a study of the stability and interactions of the many nuclides known. Ideas concerning nuclear shells were first put forward by Elasser and gratly extended by subsequent workers, particularly, Mayer.

If a nuclus posseses certain definite number- called Magic number- of protons and neutrons, it has marked stability. Such nuclei will be more stable than nuclei in their immediate neighbourhood in the same mass range. These magic numbers are
A 2, 8, 20, 40
B 2, $(6, 14, 28)$, 50, 82, 126.

The first series A can be represented by a formula $n(n+1)(n+2)/3$, where n is an integer; the second series B by $n(n^2+5)/3$. The series A is used for light nuclei, the series b is for heavy nuclei. The figures in brackets are so called semi-magic numbers; they correspond to less well-defined stabilities.

In the series A, nuclides containing $2, 8$ or 20 protons or neutrons will subsequently be more stable than their immediate neighbours. In series B, heavy nuclides containing $2, 50$ or 82 protons or neutrons or 126 neutrons will be extra stable.

1.3.1 Evidence leading to the development of nuclear shell model

The idea of the nuclear shell model was first put forward by Elasser and developed by Mayer, Jensen and others. The evidences which led to the development of such a nuclear model are discussed below.

Similar to the electronic shell model there is an ample evidence to show that nuclei having $2, 8, 20, 50, 82$ and 126 nucleons of the same kind (all protons or all neutrons) are more stable compared to those having different numbers. So it is justified that these numbers, called the magic numbers, represent something like closed nuclear shells or subshells. Since the nucleons are considered to be independent of one another, the attempts to apply Pauli's exclusion principle to complete the energy levels in a nucleus did not give the correct answer. However, the problem was solved by introducing the aspect of interactions of spin and orbital motions of nucleons.

The spin of the various nuclei and their magnetic moments are simply explained by the shell model. It is important to neutron that the magnetic moments of nuclei computed on the basis agree fairly well with the experimental values. Lastly the angular momenta of nuclei are also explained simply if the shell model is considered.

Books Suggested:
(1). Atomic and Nuclear Physics, Vol II; S. N. Ghosal
(2). Nuclear Physics, Theory and Experiment; R. R. Roy & B. P. Nigam
(3). Atomic and Nuclear Physics, An Introduction; T.A. Littlefield & N. Thorley