SEMESTER IV Paper: PHS-A-CC-4-9-TH
Lasers:Three-Level and Four-Level Lasers

Lasers: Einstein's A and B coeffcients. Metastable states. Spontaneous and Stimulated emissions. Optical Pumping and Population Inversion. Three-Level and Four-Level Lasers. Ruby Laser and HeNe Laser. Basic lasing

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1 Three-Level and Four-Level Lasers

A 'LASER' operates by the method of stimulated emission of radiation. The rate of the stimulated emission depends on the no. density of stoms in the excited state of an atomic system. The purpose is achieved by obtaining the population inversion.

Here we discuss the rate equations which governs the rate at which populations of various levels change under the action of a pump and in the presence of laser radiation. This approach provides a covenient means of studying the time dependence of the atomic population of various levels under the presence of radiation at frequencies corresponding to the different transitions of the atoms. It also gives the steady state population difference between the actual levels involved in the laser transition and allows one to study whether population inversion is achievable in a transion and, if so, what would be the minimum pumping rate required to maintain a steady population inversion for continuous wave operation of the laser.

1.1 The Three-Level System

Let us consider a three level laser system and assume that all the levels are nondegenerate. The pump is applied in the $1 \longrightarrow 3$ transition and the lasing transition is $2 \longrightarrow 1$. In this system the pump lift atoms from the level 1 into the level 3, from which they decay rapidly to the level 2 through some nonradiative process; the level 2 is required to be metastable. Thus the pump effectively transfer atoms from level 1 to level 2 through level 3. If the relaxation from level 3 to level 2 is very fast, then most of the atoms in level 3 will relax down to level 2 rather than to level 1. Since the upper pump level 3 is not one of the laser levels, level 3 can be broad level so that a broadband light source can be used efficiently as a pumping source. Also since the lower laser level is the ground level, more than 50% of the atoms from level 1 have to be lifted up to attain population inversion between level 2 and 1.

Let N_1, N_2, and N_3 represent the number of atoms per unit volume in levels $1, 2, 3$, respectively. We assume that only these three levels are populated and that the transitions take place only between these three levels. For such a case, we may write

$$N = N_1 + N_2 + N_3 \tag{1}$$

where N represents the total number of atoms per unit volume. Now the change in the population level 3 is described by

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - N_3 T_{32} \tag{2}$$

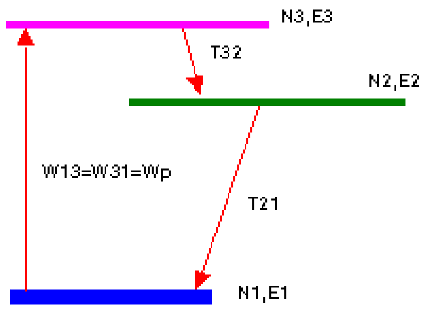

Figure 1: *The energy levels in a three level laser systems. Level* 1 *is the ground level, Levels* 2 *and* 3 *are excited levels. The pump lifts atom from level* 1 *to level* 3 *from which they decay rapidly to level* 2*. Population inversion is obtained between levels* 2 *and* 1 *and the laser oscillates at the frequency corresponding to the* $2 \longrightarrow 1 transition$*.*

where W_pN_1 represents the number of induced absorptions per unit time per unit volume which results in the $1 \longrightarrow 3$ transition: similarly W_pN_3 represents the number of stimulated emissions per unit time per unit volume associated with $3 \longrightarrow 1$ transition. The last term represents the transition caused mainly by extremely fast nonradiative process.

Further

$$T_{32} = A_{32} + S_{32} \tag{3}$$

where A_{32} represents the Einstein A coefficient (corresponding to a radiative transition) connecting levels 3 and 2, and S_{32} represents the nonradiative transition rate from levels 3 to 2.

Similarly, the rate of change of the population of level 2

$$\frac{dN_2}{dt} = W_l(N_1 - N_2) + N_3T_{32} - N_2T_{21} \tag{4}$$

where the first term on the right-hand side represents the stimulated transitions between levels 1 and 2; the second term represents spontaneous transition from level 3 to level 2 and the third term represents the spontaneous transition from level 2 to level 1. Here W_l is proportional to the Einstein coefficient B_{21} and to the energy density associated with the lasing transition $2 \longrightarrow 1$. And T_{21} represents the net spontaneous relaxation rate from level 2 to level 1. If this transition is predominantly radiative then $T_{21} \approx A_{21}$, where A_{21} represents the Einstein coefficient.

Similarly

$$\frac{dN_1}{dt} = W_p(N_3 - N_1) + W_l(N_2 - N_1) + N_2T_{21} \tag{5}$$

where the first two terms represent the stimulated transitions between levels 1 and 3 and Levels 1 and 2, respectively, and the last term represents the spontaneous transitions from level 2 to level 1.

$$\frac{dN_1}{dt} + \frac{dN_2}{dt} + \frac{dN_3}{dt} = 0 \tag{6}$$

Equations 2, 4 and 5 are referred to as the rate equations and give the rate of change of populations of the three levels in a three-level laser system in terms of W_p and W_l.

At steady state

$$N_3 = \frac{W_p}{W_p + T_{32}} N_1 \tag{7}$$

$$N_2 = \left(W_l + \frac{T_{32} W_p}{W_p + T_{32}} \right) \frac{N_1}{W_l + T_{21}} \tag{8}$$

Therefore the population difference between the levels 2 and 1 is

$$\frac{N_2 - N_1}{N} = \frac{W_p(T_{32} - T_{21}) - T_{32}T_{21}}{3W_pW_l + 2W_pT_{21} + 2T_{32}W_l + T_{32}W_p + T_{32}T_{21}} \tag{9}$$

From equation 9 it follows tha in order to obtain population inversion between levels 2 and 1 i.e, $N_2 - N_1$ to be positive, a necessary condition is that $T_{32} > T_{21}$. Since the relaxation time of atoms in levels 3 and 2 are inversely proportional to the corresponding relaxation rates, the lifetime of level 3 must be at least smaller than the life time of level 2 for attainment of population inversion. In addition, in order to achieve population inversion, a minimum pump power is required.

So

$$W_{pt} = \frac{T_{32}T_{21}}{T_{32} - T_{21}} \tag{10}$$

To obtain population inversion, W_p is required to be greater than W_pt.

2 The Four-Level Syatem

In four level laser system Level 1 is the ground level and levels 2, 3, and 4 are excited levels of the system. Atoms from the level 1 are excited by a pump to level 4, from which the atoms decay very rapidly through some nonradiative transition to level 3. Level 3 is a metastable level having a long lifetime. This level forms the upper laser level and level 2 forms the lower laser level. The lower laser level must have a very short lifetime so that the incoming atoms from level 3 relax down immediately from level 2 to level 1, ready for being pumped to level 4. If the rate of relaxation of atoms from level 2 to level 1 is faster than the rate of arrival of atoms into level 2, one can obtain population inversion between level 3 and 2 even for very small pump powers. The amount of population inversion between levels 3 and 2 would depend on pumping rate.

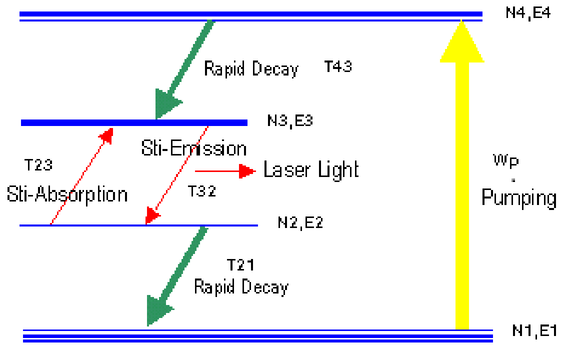

Figure 2: *The energy levels in a four level laser systems. Level* 1 *is the ground level, Levels* 2, 3 *and* 4 *are excited levels. The pump lifts atom from level* 1 *to level* 4 *from where they make a fast transition to level* 3*, which has a slow relaxation rate.Laser action is obtained between levels* 3 *and* 2 *when* N_3 *becomes greater than* N_2*. The atoms dropping to* E_2 *make another fast nonradiatiive transition to the ground level* 1.

The level 4 could be a collection of large number of levels or a broad level. Then an optical pump emitting radiation over a band of frequencies can efficiently excite atoms from level 1 to level 4. Also level 4 can not be upper laser level because the upper laser level is required to be narrow. In addition the lower laser level 2 is required to be sufficiently above the ground level 1 so that at ordinary temperatures the population of level 2 is negligible. If the level 2 is not sufficiently above level 1 then the pumping power required for achieving population inversion would be much more. At the same time the population of level 2 may also be lowered by lowering the temperature of the system.

Now the rate equations for the various levels of the four level system is given as following:

Let N_1, N_2, N_3 and N_4 represents the population per unit volume of the levels 1, 2, 3 and 4 respectively. The change in population of level 4 is given by

$$\frac{dN_4}{dt} = W_p(N_1 - N_4) - T_4 N_4 \tag{11}$$

where $W_p(N_1 - N_4)$ represents the net rate of stimulated transitions between level 1 and 4 caused by the pupm beam, T_4 is the net relaxation rate from level 4 to the lower levels 3, 2, and 1,so

$$T_4 = T_{43} + T_{42} + T_{41} \tag{12}$$

Here the T's represents the total relaxation rates (both radiative and nonradiative). Also T_{43} is usually much greater than T_{42} and T_{41}, so that most of the atoms pumped to level 4 drop down to level 3. The first term represents the rate at which atoms are being pumped from level 1 to level 4 and the second term represents the rate at which atoms decay from level 4.

Similarly the rate equation for N_3 would be

$$\frac{dN_3}{dt} = T_{43}N_4 + W_l(N_2 - N_3) - T_3N_3 \tag{13}$$

where $T_3 = T_{32} + T_{31}$. The first term represents the rate at which atom drop down from level 4 into level 3, the second term represents the rate of stimulated transitions from level 2 to level 3 due the presence of laser radiation, and the third term represents the rate of loss of atoms from level 3 to level 2 and 1 through spontaneous transitions.

Similarly the rate equations for N_2 and N_1 would be

$$\frac{dN_2}{dt} = T_{42}N_4 + W_l(N_3 - N_2) - T_{21}N_2 + T_{32}N_3 \tag{14}$$

$$\frac{dN_1}{dt} = W_p(N_4 - N_1) + T_{41}N_4 + T_{31}N_3 + T_{21}N_2 \tag{15}$$

It can be seen that

$$\frac{dN_1}{dt} + \frac{dN_2}{dt} + \frac{dN_3}{dt} + \frac{dN_4}{dt} = 0 \tag{16}$$

which is consistent with the requirement that the total no of atms must be constant.

$$N = N_1 + N_2 + N_3 + N_4 \tag{17}$$

At steady state

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dN_3}{dt} = \frac{dN_4}{dt} = 0 \tag{18}$$

Therefore

$$N_4 = \frac{W_p}{2W_p + T_4}(N - N_2 - N_3) \tag{19}$$

Using Eqns. 17, 19 and Eqns. 13 14 and 18, we obtain

$$N_3\left[(W_l + T_3)\left(\frac{2W_p + T_4}{W_p}\right) + T_{43}\right] + N_2\left[T_{43} - \frac{W_l(2W_p + T_4)}{W_p}\right] = T_{43}N \tag{20}$$

$$N_3\left[T_{42}-(T_{32}+W_l)\left(\frac{2W_p+T_4}{W_p}\right)\right]+N_2\left[(T_{21}+W_l)\left(\frac{2W_p+T_4}{W_p}\right)+T_{42}\right]=NT_{42} \tag{21}$$

We now solve the above two equation to obtain the population difference between level 3 and 2, namely, $N_3 - N_2$. We have

$$\frac{N_3-N_2}{N}=(T_{21}T_{43}-T_3T_{42}-T_{32}T_{43})\times$$
$$\left\{T_{43}(T_{21}+T_{32})+T_3T_{42}+\left(\frac{2W_p+T_4}{W_p}\right)T_3T_{21}+W_l\left[2(T_{42}+T_{43})+\left(\frac{2W_p+T_4}{W_p}\right)(T_{31}+T_{21})\right]\right\}^{-1} \tag{22}$$

This equation gives the steady state population difference as a function of the pump power, the laser power, and the lifetimes of the various states involved in the four level system. In most four-level systems, the atom from level 4 relax primarily to level 3 and hence $T_{42}, T_{41} << T_{43}$. Also the atoms from level 3 relax primarily from level 2, i.e., $T_3 \approx T_{32}$. Under these approximations, we see that in order to obtain a population inversion, we must have

$$T_{21} > T_{32} \tag{23}$$

Under such a condition, the creation of population inversion betweens level 3 and 2 is independent of the pumping power but the magnitude of the population inversion depend on W_p.

At and below threshold conditions, $W_l \approx 0$ so the approximate equation for the population difference is given by

$$\frac{N_3-N_2}{N}=\frac{(1-\beta)W_pT_{43}/T_4T_3}{1+[(1+\beta)+2T_3/T_{43}]\,W_pT_{43}/T_4T_3} \tag{24}$$

where

$$\beta=\frac{T_{32}}{T_{21}}+\frac{T_3T_{42}}{T_{21}T_{43}} \tag{25}$$

For good laser action one must have $T_3 << T_{43}$ and $T_{21} >> T_{32}$ so that $\beta \to 0$. Also $T_4 \approx T_{43}$. Therefore

$$\frac{N_3-N_2}{N}=\frac{W_p/T_3}{1+W_p/T_3} \tag{26}$$

If we compare the steady state population difference between three level and four level laser system we can see that the inversion can be much more easily obtained in a four level scheme as compared to a three level system.

Book Suggested:Lasers; Theory and Applications, K. Tyagrajan and A. K. Ghatak