Elements of Modern Physics

Lasers: Einstein's A and B coeffcients. Metastable states. Spontaneous and Stimulated emissions. Optical Pumping and Population Inversion. Three-Level and Four-Level Lasers. Ruby Laser and HeNe Laser. Basic lasing

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1 Einstein's A and B coefficients

An atom in an excited state can make a radiative transition to the ground state with the emission of photons in two ways:
(i). Spontaneous Emission, which takes place in absence of any external agency and

(ii). Stimulated Emission, which is caused by the presence of an e.m. radiation of proper frequency.

In the case of stimilated emission, the excited atom is irradiated with another photon having the same energy as that of the spontaneous emitted photon. The radiation triggers the release of the photon, producing two, where there would be one in case of spontaneous radiation. The incident radiation is thus amplified.

we consider an atom having two energy states 1 and 2, having energy values E_1 and E_2 respectively such that $E_2 > E_1$. Let N_1 and N_2 be the no. of atoms per unit volume (called the population of the particular level) of staes 1 and 2 respectively. Let $U_\nu d\nu$ represents the energy density or the intensity of the radiation field in the frequency interval ν to $(\nu + d\nu)$. For the absorption process the rate of excitation (i.e. the no. of absorption per unit volume per unit time) to the energy level E_2 from the level E_1 is proportional to
(i). the population of the level E_1 and
(ii). the intensity of radiation field.

E_2	$\uparrow$	$\vert$	N_2
absorption	$\vert$	$\vert$	*emission*
E_1	$\vert$	$\downarrow$	N_1

Therefore

$$\frac{dN_2}{dt}|_{abs} = B_{12} N_1 U(\nu), \tag{1}$$

where B_{12} is a constant. The spontaneous emission process, which is a random process, depends only on the population N_2 of the level 2 and hence

$$\frac{dN_2}{dt}|_{spt.emi} = -A_{21} N_2, \tag{2}$$

where A_{21} is a constant

For the stimilated emission, the probality of transition from the level 2 to the level 1 is proportional to
(i). the intensity of the radiation field and
(ii). the poulation N_2 of the state 2.

Therefore the rate of stimilated emission per unit volume

$$\frac{dN_2}{dt}|_{sti.emi.} = B_{21}N_2U(\nu), \tag{3}$$

where B_{21} is a constant.

Effectively

$$\begin{aligned}\frac{dN_2}{dt} &= B_{12}N_1U(\nu) - A_{21}N_2 - B_{21}N_2U(\nu) \\ &= -A_{21}N_2 + (B_{12}N_1 - B_{21}N_2)U(\nu)\end{aligned} \tag{4}$$

At thermal equilibrium at $T^\circ K$,

$$\frac{dN_2}{dt} = 0 \tag{5}$$

Therefore

$$\begin{aligned}U(\nu) &= \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2} \\ &= \frac{A_{21}}{B_{12}\frac{N_1}{N_2} - B_{21}}\end{aligned} \tag{6}$$

If ν be the frequency of the radiation corresponding to transition from the level 2 to the level 1, we have $h\nu = E_2 - E_1$.
Again, at the equilibrium temp $T^\circ K$ N_1 and N_2 are related by

$$N_2 = N_1 e^{-(E_2 - E_1)/kT} \tag{7}$$

, where k is the Boltzmann's constant.

Therefore

$$\frac{N_1}{N_2} = e^{h\nu/kT} \tag{8}$$

From (5)

$$\begin{aligned}U(\nu) &= \frac{A_{21}}{B_{12}e^{h\nu/kT} - B_{21}} \\ &= \frac{A_{21}/B_{12}}{e^{h\nu/kT} - \frac{B_{21}}{B_{12}}}\end{aligned} \tag{9}$$

now from Planck's law, we have

$$U(\nu) = \frac{8\pi h\nu^3}{c^3}.\frac{1}{e^{h\nu/kT}-1} \tag{10}$$

Comparing (9) and (10), we find that

$$\frac{B_{21}}{B_{12}} = 1 \tag{11}$$

$$\longrightarrow B_{12} = B_{21} \tag{12}$$

and

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h\nu^3}{c^3} \tag{13}$$

Therefore

$$A_{21} = \frac{8\pi h\nu^3}{c^3}B_{21} \tag{14}$$

These A and B are known as Einstein's co-efficients.

1.1 Stimulated Emission

In an atomic system the excited atoms undergo spontanous transition to the lower energy states. If an exernal radiation of proper frequency is incident on the system, the external radiation induces the excited atoms to make transitions to the lower energy states. Thus apart from spontaneous emission of radiation, another radiation is achieved in this case. This emission of radiation triggered by the external radiation is called the stimulated emission.

1.2 Population Inversion

Usually the number density of atoms of an atomic system in the ground state remains larger than in the excited state. If by proper pumping, we obtain a situation so that the number density of atoms in the excited state becomes larger than that of the lower or normal state, population inversion is said to be achieved.

1.3 Basic condition for Lasing action:

(1). The atomic system to be used for lasing action must have more than one quantized states.
(2). Population inversion must be achieved. For this purpose a continuous pump field

must operate to raise the atoms to an excited state.
(3). To maintain population inversion, a metastable state having larger life time must be created where the atoms will build up.
(4). Coherency condion must be achieved, otherwise light amplification could not take place
(5) The signal frequency or the probe frequency must be strictly monochromatic and equal to the frequency of the level difference between which population inversion has been created.