SEMESTER IV
Paper: PHS-A-CC-4-9-TH
Radioactivity: Beta and Gamma Decay

Radioactivity: stability of the nucleus; Law of radioactive decay; Mean life and half-life; Alpha decay; Beta decay- energy released, spectrum and Pauli's prediction of neutrino; Gamma ray emission, energy-momentum conservation: electron-positron pair creation by gamma photons in the vicinity of a nucleus.

1 Radioactivity:

1.1 Beta Decay

The β decay occurs between a pair of isobars, i.e., two atoms with the same mass number A but different atomic numbers. If a β disintegration takes place from a nucleus of mass number A and atomic number Z, the resulting nucleus is a mass number A and atomic number $(Z+1)$. Hence the energy of the β particle is

$$Q_\beta = {}^{A}_{Z}M_n - \left[{}^{A}_{Z+1}M_n + m_e\right],$$

where M_n represents the mass of a nucleus and m_e now signifies the mass of an electron. Both the parent and daughter nuclei are assumed to be in their ground energy states.

$$Q_\beta = \left[{}^{A}_{Z}M_n + Zm_e\right] - \left[{}^{A}_{Z+1}M_n + Zm_e + m_e\right]$$

$$Q_\beta = {}^{A}_{Z}M - {}^{A}_{Z+1}M, \tag{1}$$

where ${}^{A}_{Z}M$ is the atomic mass of the isobars of mass number A and atomic number Z while ${}^{A}_{Z+1}M$ is the atomic mass of mass number A and atomic number $(Z+1)$. Now the β particles which are fermions, have a spin angular momentum of $\hbar/2$. So, when they are emitted from the nucleus, they should change their nuclear spin by half. but due to β decay, a nuclei which had an integral spin, either remained with the same spin or an integral change in spin was involved.

In order to explain this anomally as well as the paradox of the continuous β ray spectrum, Pauli suggested that another neutral particle which has a negligible mass compared to that of an electron and a spin angular momentum$\hbar/2$ is emitted along with the β particles. This particle was called 'neutrino(ν) by him. Because of the comparatively large mass of the nucleus, the recoil energy of the nucleus is very small and nearly all the kinetic energy is shared between the β particles and the 'neutrino'. Since the neutrino can take up any value of the emitted energy upto a finite limit, the emitted β particles is associated with any possible amount of energy as desired by the continuous β ray spectrum (Figure 1). Further, the vector sum of the spin (intrinsic) angular momenta of the neutrino and the β particle will be either zero or one in units of $\hbar$. Thus the conservation of angular momenta or spin in β decay was also achieved. The basic β decay process can thus be written as

$$\begin{aligned}
\beta^- decay; \quad & {}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e + \bar{\nu} \quad [{}^{1}_{1}p \rightarrow proton] \\
\beta^- + decay; \quad & {}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{1}e + \nu \quad [{}^{1}_{0}n \rightarrow neutron] \\
& k^- capture \quad {}^{1}_{1}p + e^- \rightarrow {}^{1}_{0}n + \nu
\end{aligned}$$

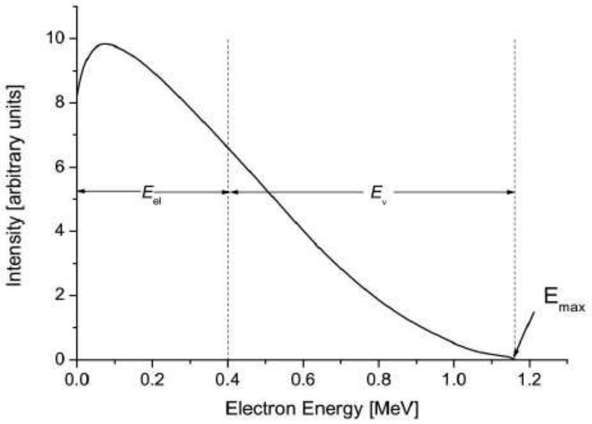

Figure 1: *β ray Spectrum*

A satisfactory theory of β- disintegration was ggiven by Fermi. The theory states that for allowed transitions, the spectral distribution is of the form

$$N(p)dp = Cp^2(E_m - E)^2F(Z,p)dp \tag{2}$$

where N(p) is the number of β particles emitted with momenta lying between p and $p+dp$; C is a constant for the nuclei involved in the β decay; E_m is the maximum energy of the spectrum; E is the kinetic energy of the β particles; $F(Z,p)$ is the Coulomb correction factor that takes into account the Coulomb interaction between the nuclear particles and β particles and Z is the atomic number of the product nucleus. Equation 2 can be written as

$$\sqrt{\frac{N(p)}{p^2F(Z,p)}} = K(E_m - E), \tag{3}$$

where K is a new constant. Thus if $\sqrt{\frac{N(p)}{p^2F(Z,p)}}$ is plotted against the energy(E) of the β particles a straight line will be obtained with an energy intercept at E_m. This is known as 'Kurie Plot'.

2 Gamma Ray Emission

A nucleus can exist in states whose energy is higher than that its ground state, just as an atom can. Excited nuclei return to their ground states by emitting photons whose energies correspond to the energy differences between the various initial and final states in the transitions involved. The photons emitted by nuclei range in energy up to several MeV, and are traditionally called gamma rays.

One example of gamma ray production due to radionuclide decay is the decay scheme for cobalt-60, as illustrated in the Figure (2). First, Co-60 decays to excited Ni-60 by beta decay emission of an electron of 0.31 MeV. Then the excited Ni-60 decays to the ground state by emitting gamma rays in succession of 1.17 MeV followed by 1.33 MeV. This path is followed 99.88 % of the time.

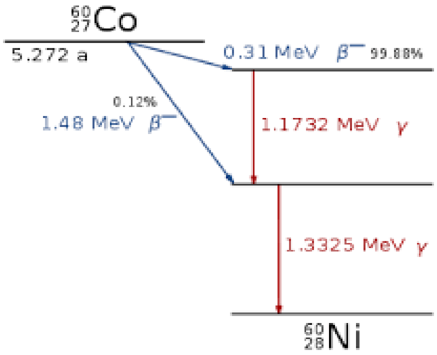

Figure 2: γ *ray Spectrum*

$$^{60}_{27}Co \rightarrow {}^{60}_{28}Ni^{*} + e^{-} + \bar{\nu}_e + 1.17MeV$$

$$^{60}_{28}Ni^{*} \rightarrow {}^{60}_{28}Ni + \nu + 1.33MeV$$

2.1 Interaction of γ rays with matter:

The γ rays having an energy range from 0.1 MeV to 25 MeV interact with matter by three important process. Those are
(a). The phhotoelectric effect which is an inelastic process in which the photon disappear ejecting an inner orbital electron from the absorber atom;
(b). The Compton Effect which is an elastic scattering process in which photon imparts energy to a free electron and itself get scattered with larger wavelength and
(c). Pair production in which the photon disappears and is converted to electron and positron pair.

Pair production is possible only if the energy of the γ photon is not less than the sum of the rest mass energies of the electron-positron pair. The pair production process can not take place in free space and being an electromagnetic process usually occurs in the intense electric field in the vicinity of the nucleus of the absorber. The nucleus recoils in this process conserving momentum, though the kinetic energy carried away by it is small due to its large mass compared with that of electron. And the photon energy, if any, in excess of $2m_ec^2$ is shared as K.E. by the product particles, where m_e is the rest mass of electron or positron.

At very high photon energies greater than $2m_ec^2$, pair production process predominates. It can be shown that the pair production absorption coefficient μ_p is proportional to nZ^2 for a given γ photon energy and increases with this energy, starting with the value zero when the photon energy is just on the lower side of $2m_ec^2$. Here n is the number of atoms in unit volume of the absorber element. In the region of γ photon energies less than $2m_ec^2$, the compton effect predominates and it decreases slowly as the photon energy increases. When the energy of the photon is much greater than the rest mass energy of the electron, it is found that the Compton scattering coefficient μ_s is proportional to nZ. For photon energies lower than $0.5MeV$, the photoelectric effect occurs with the bound atomic electron and not with the free electron. Taking the ejection of bound electrons of the photoelectrons from K shell only, non-relativistic calculations show that the photoelectric coefficient μ_{pe} is found to be proportional to nZ^5.

In the process (a) and (c), the photon vanishes and in (b) its energy is lost to some extent only. Thus unlike in the cases α, β and other charged particles which lose their energy in their interaction with matter through several small steps in the γ rays, the absorption of γ photon is a single step one. This makes the absorption of γ rays by matter an exponential process with two well defined maximum range. There is a finite probability for a γ photon to emerge through an absorber whatever its thickness without suffering any of the interactions mentioned above. In fact the intensity of γ ray photon after traversing a distance x of the absorbing material is given by

$$I_x = I_0 e^{-\mu_x}, \tag{4}$$

where I_0 is the intensity of the beam incident normally on the absorbing material.
$\therefore \mu_x = (\mu_p + \mu_s + \mu_{pe})$ is the linear absorption coefficient of the γ radiation.

2.2 Electron-positron pair production by γ rays

We have seen that some artificially produced radioactive nuclei decay by positron emission. The positron is the antiparticle of the electron, having the same mass and spin (1/2) as the electron, carrying a charge equal and opposite to that of the electron.

The existence of the positron was predicted by P.A.M. Dirac. Dirac tried to develop a quantum mechanical theory of the electron which took into account the relativistic relation between the energy and momentum $E^2 = p^2c^2 + {m_0}^2c^4$, where E is the total energy and p is the momentum of the particle; m_0 is the rest mass of the particle. Dirac sought a linear relationship between E and p whose operator representations are

$$\hat{p} \to \frac{\hbar}{i}\nabla, \hat{E} \to \hbar\frac{\partial}{\partial t}$$

The equations which dirac postulated for a free (relativistic) particle has the form:

$$\hbar\frac{\partial\psi}{\partial t} = \hbar c\alpha.\nabla\psi + \beta mc^2\psi \tag{5}$$

where $m = \frac{m_0}{\sqrt{1-\beta^2}}$; m is the relativistic mass. The three components of α and β anticommute and hence cannot be numbers. They are 4×4 matrices. So the wave-function ψ must have four components.

Dirac interpreted two of the components of ψ to represent the two possible spin states of a spin 1/2 particle(electron) with spin 'up' and 'down'. The other two solutions are the two spin states of another spin 1/2 particle, called the antielectron. According to Dirac the two particles would have opposite charges, but the same mass, spin, etc.

Thus Dirac theory predicted that the particles could have an intrinsic spin and would also have antiparticles. This theory also tells that the total energy of the electron, inclusive of its rest energy, can be both positive and negative: $E = \pm\sqrt{p^2c^2 + {m_0}^2c^4}$.. This relation shows that the total energy of the elctron can be either greater than $+m_0c^2$ or less than $-m_0c^2$, where m_0 is the rest mass of the electron.

An electron of positive energy $E > m_0c^2$ is the electron that is actually found in the physical world. And all the negative energy states are completely filled with electrons. As the electrons obey Pauli's exclusion principle, no two electrons can occupy the same state. So no electron from a positive energy state can make transition to any of the negative energy states, since the latter are all filled with electrons.

However, if by some means an electron is removed from a negative energy state, then a vacancy or hole is created in the sea of negative energy states filled with electrons. Since such a hole is created due to the absence of a negatively charged electron, it will behave as apositively charged particle with charge $+e$. Further the absence of a particle in the state of negative energy $-E$ and of negative momentum $-p$ will manifest as a particle of positive energy $+E$ and positive momentum $+p$. Thus the hole created in the negative energy state will appear as a positively charged particle in the positive energy state and hence should be observable in our physical world.

To produce a positron, it is necessary to create a vacancy in a negative energy state of electrons. Since all negative energy states are already filled with electrons, a vacancy in one of these states can be created only if the electron occupying it is transferred to a vacant positive energy state, since Pauli's exclusion principle forbids its transfer to a state of negative energy. The minimum energy needed for such transition is $2m_0c^2 = 1.022MeV$. This energy can be given to the electron in the negative energy state by bombarding it with γ ray photons of energy $E_\gamma > 2m_0c^2$. On absorbing the photon, the electron is transferred to a vacant positive energy state and then it behaves like a normally obsevable electron. The hole created in the negative energy stae, of course, behave like a positron. Thus the action of the γ ray of energy greater than $2m_0c^2$ is to create simultaneously an electron-positron pair.

Book Suggested:
Concept of Modern Physics: Arthur Beiser
Atomic and Nuclear Physics: S. N. Ghosal
Image credit: Wikipedia

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