SEMESTER IV
Paper: PHS-A-CC-4-9-TH
Radioactivity

Radioactivity: stability of the nucleus; Law of radioactive decay; Mean life and half-life; Alpha decay; Beta decay- energy released, spectrum and Pauli's prediction of neutrino; Gamma ray emission, energy-momentum conservation: electron-positron pair creation by gamma photons in the vicinity of a nucleus.

1 Radioactivity:

1.1 Radioactive decay

Radioactivity was discovered in 1896 by Antoine Becquerel. From the perspective of classical physics three features of radioactivity are extraordinary as described below:

(i). When a nucleus undergoes α or β decay, its atomic number Z changes and it becomes the nucleus of different element.

(ii). The energy liberated during radioactive decay comes from within individual nuclei without external excitation, unlike the case of atomic radiation.

(iii). Radioactive decay is a statistical process that obeys the laws of chance. No cause effect relationship is involved in the decay of a particular nucleus, only in a certain probability per unit time. Classical physics cannot account for such behavior, although fits natuarally into the framework of quantum physics.

The early experiments, among them Rutherford and his coworkers, distinguished three components in the radiations from radionuclides. These components were called alpha, beta and gamma, which were eventually identified as ${}^4_2He$ nuclei, electrons and high-energy photons respectively. Later, positron emission and electron capture were added to the list of the decay modes. examples of the nuclear transformations that accompany the various decays are given the Table 1.

Table 1: Radioactive Decay

Decay	Transformation	Example
Alpha decay	${}^A_ZX \rightarrow {}^{A-4}_{Z-2}Y + {}^4_2He$	${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^4_2He$
Beta decay	${}^A_ZX \rightarrow {}^A_{Z+1}Y + e^-$	${}^{14}_6C \rightarrow {}^{14}_7N + e^-$
Positron emission	${}^A_ZX \rightarrow {}^A_{Z-1}Y + e^+$	${}^{64}_{29}Cu \rightarrow {}^{64}_{28}Ni + e^+$
Electron capture	${}^A_ZX + e^- \rightarrow {}^A_{Z-1}Y$	${}^{64}_{29}Cu + e^- \rightarrow {}^{64}_{28}Ni$
Gamma decay	${}^A_ZX^\star \rightarrow {}^A_ZX + \gamma$	${}^{87}_{38}Sr^\star \rightarrow {}^{87}_{38}Sr + e^-$

The * denotes an excited nuclear state and γ denotes a gamma-ray photon.

1.2 Activity

The **activity** of a sample of any radioactive nuclide is the rate at which the nuclei of its constituent atoms decay. If N is the number of nuclei present in the sample at a certain time, its activity R is given by

$$R = -\frac{dN}{dt} \tag{1}$$

The SI unit of activity is named after Becquerel. 1 becquerel = 1 Bq = 1 decay/s.

The traditional unit of activity is the **curie**(Ci), which was originally defined as the activity of 1 g of radium, ${}^{226}_{88}Ra$. Because the precise value of the curie changed as the method of measurement improved, it is now defined arbitrarily as
1 curie = 1 $Ci = 3.70 \times 10^{10}$ decays/s= 37 GBq.

2 Law of Radioactive Decay

Rutherford and Soddy made an experimental study of the radioactive decay of various radioactive materials and found that the decay of all radioactive materials is governed by the same general law. According to the law the atoms of radioactive element are transformed into other atoms as a result of α or β disintegration. The rate of transformation of the radiactive atoms at any instant, depends on the number of atoms present in the sample.

Consider the radioactive disintegration of the atoms of an element P into those another element Q written as $P \rightarrow Q$. The number of atoms of P decreases with time, while that of Q increases with time. If N be the number of atoms of P at any instant, then the rate of change dN/dt of N with time is proportional to N so that

$$\frac{dN}{dt} \propto N$$
$$\frac{dN}{dt} = -\lambda N \tag{2}$$

Here λ is a constant, known as the disintegration constant or decay constant. Integrating the above equation we get

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$N = N_0 e^{-\lambda t} \tag{3}$$

2.1 Half-Life

Suppose that the number of radioactive atoms is reduced to half in the time τ known as the half-life of disintegration. So when $t = \tau, N = N_0/2$ so that we get

$$\frac{N_0}{2} = N_0 e^{-\lambda\tau}$$

which gives

$$\tau = \frac{ln2}{\lambda} = \frac{0.693}{\lambda} \tag{4}$$

2.2 Mean-life

The mean lifetime of a nuclide is the reciprocal of its decay probality per unit time.

$$\bar{T} = \frac{1}{\lambda} \tag{5}$$

Hence

$$\bar{T} = \frac{1}{\lambda} = \frac{\tau}{0.693} = 1.44\tau \tag{6}$$

$\bar{T}$ is nearly half again more more than τ.

Since the activity of a radioactive sample is defined as

$$R = -\frac{dN}{dt}$$

we see that

$$R = \lambda N_0 e^{-\lambda t}$$

This agrees with the activity law $R = R_0 e^{-\lambda t}$ if $R_0 = \lambda N_0$ or, in general, if **Activity** $R = \lambda N$.

3 Alpha Decay

Because the attractive forces between nucleons are of short range, the total binding energy in a nucleus is approximately proportional to its mass number A, the number of nucleons it contains. The repulsive electric forces between protons, however are of unlimited range, and the total disruptive energy in a nucleus is approximately proportional to Z^2. Nuclei which contain 210 or more nucleons are so large that the short range nuclear forces that hold them together are barely able to counterbalance the mutual repulsion of their protons. Alpha decay occurs in such a nuclei as a means of increasing their stability by reducing their size.

At the time of α emission, the emitting nucleus is normally at rest, having zero momentum. Hence the total momentum of the final products in the α disintegration must always be zero. Since the α particles are emitted with a definite kinetic energy, they must have a definite momentum each. Hence the residual nucleus left after the α disintegration must have recoil momentum, equal and opposite to the α momentum. Thus the residual nucleus must have a certain amount of kinetic energy. Since these nuclei are usually much heavier than the α particles, their kinetic energy must be small compared to that of the latter ($E_g = p^2/2M_1$).

The α disintegration of a nucleus X of mass number A and atomic number Z can be written as

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

Here Y is the residual nucleus of mass number $A-4$ and atomic number $Z-2$.

If the mass of the α particle ($^{4}_{2}He$) and the residual be M_α and M_1 respectively, and v_α and v_1 their respective velocities then conservation of momentum requires that

$$M_\alpha v_\alpha = M_1 v_1$$

If Q is the α disintegration energy, which is the total energy released in the disintegration process, we can write

$$\begin{aligned} Q &= \frac{1}{2}M_\alpha {v_\alpha}^2 + \frac{1}{2}M_1 {v_1}^2 \\ &= \frac{1}{2}M_\alpha {v_\alpha}^2 + \frac{1}{2}\frac{{M_\alpha}^2 {v_\alpha}^2}{M_1} \\ &= \frac{1}{2}M_\alpha {v_\alpha}^2 \left(1 + \frac{M_\alpha}{M_1}\right) \end{aligned}$$

The kinetic energy of the α particle is

$$E_\alpha = \frac{1}{2}M_\alpha {v_\alpha}^2.$$

We then get

$$Q = E_\alpha . \frac{M_1 + M_\alpha}{M_1}$$

Since the masses of the nuclei in the unit of atomic masses are close to their mass numbers, we can write $M_1 \approx A-4$ and $M_\alpha \approx 4$, so that

$$Q = E_\alpha . \frac{A}{A-4} \tag{7}$$

Since E_α can be measured experimentally, Q can be determined from Eq. 7. Obviously $Q > E_\alpha$.

3.1 Range of the α particles: Geiger-Nuttall Law

The monoenergetic α particles from a given source can travel through a definite maximum distance from the source within a given substance. This distance is known as the range of the α particles. Measured in the unit of length, the range is very small in a solid or in a liquid ($\sim 10^{-3}mm$ for α energy of few Mev). Because of the low density of gas, the range in a gas is relatively much longer (few cm). In the case of a gas, the range depends on the temperature and the pressure of the gas. With increase of pressure, the range decreases, while it increases with increase of temperature.
From the measured values of ranges and energies of the α particles, these two quantities have been related by

$$R = aE^{3/2} \tag{8}$$

This emperical relationship, valid in a limited energy range, is known as Geiger's law. For E in Mev and R in metre, the constant $a = 3.15 \times 10^{-3}$.
If v be the velocity then since $v \propto \sqrt{E}$, we can write

$$R = bv^3 \tag{9}$$

where b is another constant: $b = 9.416 \times 10^{-24}$. In the case of a solid, the range R_s (in metre) is related to the range R in air as follows:

$$R_s = \frac{0.312RA^{1/2}}{\rho} \tag{10}$$

where ρ is the density of the solid of mass number A.

Geiger and Nuttall (1911) discovered an emperical relationship between the ranges of α particles and the disintegration constants of the naturally α active substances emitting them. This is known as Geiger-Nuttall law which can be expressed as

$$log\lambda = A + B\ logR \tag{11}$$

A and B are constants. According to this law, the α particles emitted by substances having larger disintegration constants have longer ranges and vice-versa.

Equation (11) shows that the graph of $log\lambda$ and $logR$ is a straight line with a slope B. For different radioactive series, different straight lines are obtained, which is parallel to one another, so that B is the same for all of them.

Using equation (11), it is possible to determine the disintegration constant and hence the half-life τ of a radioactive substance if the range of α particles emitted by it is known accurately.

Since the range $R \propto E^{3/2}$, the Geiger-Nuttall law can also be written as

$$log\lambda = C + D\ logE \tag{12}$$

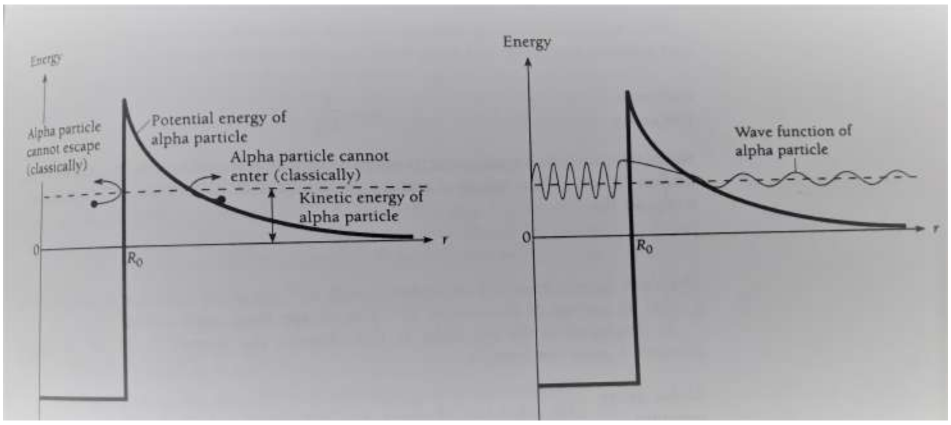

Figure 1: *Coulomb potential barrier for α disintegration: Image Courtesy:Arthur Baiser, Concept of Modern Physics*

where C and D are two other constants.

Since the half-life $\tau = ln2/\lambda$, Geiger-Nuttall law can also be expressed by relating the variation of $log\tau$ with $logR$ or $logE$. In this case we get straight line graphs, but with negative slopes.

3.2 Theory of α disintegration

The α disintegration of heavy nuclei shows that within such nuclei, two protons and two neutrons somes form a cluster known as α particle. As. long as an α particle is within the nucleus, it must be acted upon by this strong short range specifically nuclear attractive force. However once it is outside the nucleus, the Coulomb repulsion due to the residual nucleus of positive charge $(Z-2)$ acts on it, where $+Ze$ is the charge of the original nucleus. Thus the potential energy of an α particle at various distances from the centre of the atom has the appearence shown in Fig (1).

Outside the nucleus, the Coulomb potential energy

$$V_c = \frac{2(Z-2)e^2}{4\pi\epsilon_0 r} \tag{13}$$

is positive, where ϵ_0 is the permittivitty of vacuum. For $r < R$, the nuclear radius this suddenly changes into an attractive potential. The exact nature of the potential is not known. It is a strong short range attractive potential so that it must have a very steep positive slope.

The transition from the repulsive Coulomb potential to the attractive potential in the nucleus take place at the nuclear surface $r = R$ where the Coulomb potential has the highest value.

$$V_s = \frac{2(Z-2)e^2}{4\pi\epsilon_0 r} \tag{14}$$

Figure (1) is a plot of the potential energy U of an α particle as a function of its distance r from the centre of a certain heavy nucleus. The height of the potential barrier is about 25 Mev, which is equal to the work that must be done against the repulsive electric force to bring an α particle from infinity to a position adjacent to the nucleus but just outside the range of its attractive forces. We may therefore regard an α particle in such a nucleus as being inside a box whose walls require an energy of 25 Mev to be surmounted. However, decay α particles have energies that range from 4 to 9 Mev, depending on the particular nuclei involved-16 to 21 Mev short of energies needed for escape.

In classical physics, an α particle whose kinetic energy is less than the height of the potential barrier around a nucleus cannot enter or leave the nucleus, whose radius is R_0 (Left image of Figure (1)). But in quantum physics, such an α particle can tunnel through the potential barrier with a probability that decreases with the height and thickness of the barrier (Right image of Figure(1)).

Although α decay is inexplicable classically, quantum mechanics provides a straightforward explanation. The theory of α decay, developed independently in 1928 by Gamow and by Gurney and Condon. The basic notion of this theory are:

(1). An α particle may exiist as an entity within a heavy nucleus.

(2). Such a particle is in constant motion and is held in the nucleus by a potential barrier.

(3).There is a small but definite likelihood that the particle may tunnel through the barrier each time a collision with it occurs.

According to the last assumption the decay probability per unit time λ can be expressed as

$$\textit{Decay constant} \qquad \lambda = \nu T \tag{15}$$

Here ν is the number of times per second an α particle within a nucleus strikes the potential barrier around it and T is the probability that the particle will be transmitted through the barrier.

In quantum mechanics, the particle is represented by a wave, obeying Schödinger equation. One can write down the wave equation for the different regions by substituting the corresponding potentials acting on the α particle in theses regions. If theses equations are solved with appropriate boundary conditions, then it is found that an α particle initially inside the nucleus has a finite probability of coming out of it. Therefore the tunnel theory for the decay constant λ gives the formula

$$log_{10}\,\lambda = log_{10}\left(\frac{v}{2R_0}\right) + 1.29Z^{1/2}{R_0}^{1/2} - 1.72E^{-1/2} \tag{16}$$

Here v is the α particle velocity in m/s and E its energy in Mev, R_0 is the nuclear radius in fermi, and Z is the atomic number of the daughter nucleus.

Book Suggested:
Concept of Modern Physics: Arthur Beiser
Atomic and Nuclear Physics: S. N. Ghosal

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