Lasers:Three-Level Lasers: Rate Equation

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Three-Level and Four-Level Lasers: Rate equation

- A 'LASER' operates by the method of stimulated emission of radiation. The rate of the stimulated emission depends on the no. density of stoms in the excited state of an atomic system. The purpose is achieved by obtaining the population inversion.
- Rate equations governs the rate at which populations of various levels change under the action of a pump and in the presence of laser radiation.
- This approach provides a covenient means of studying the time dependence of the atomic population of various levels under the presence of radiation at frequencies corresponding to the different transitions of the atoms.
- It also gives the steady state population difference between the actual levels involved in the laser transition and allows one to study whether population inversion is achievable in a transion and, if so, what would be the minimum pumping rate required to maintain a steady population inversion for continuous wave operation of the laser.

The Three-Level System

- Consider a three level laser system
- assume that all the levels are nondegenerate
- The pump is applied in the $1 \longrightarrow 3$ transition and the lasing transition is $2 \longrightarrow 1$
- Pump lift atoms from the level 1 into the level 3, from which they decay rapidly to the level 2 through some nonradiative process; the level 2 is required to be metastable.

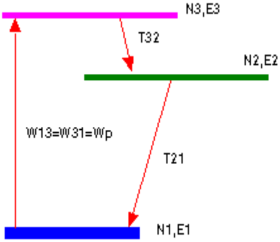

Figure: The energy levels in a three level laser systems. Level 1 is the ground level, Levels 2 and 3 are excited levels. The pump lifts atom from level 1 to level 3 from which they decay rapidly to level 2. Population inversion is obtained between levels 2 and 1 and the laser oscillates at the frequency corresponding to the $2 \longrightarrow 1$transition.

The Three-Level System

- The pump effectively transfer atoms from level 1 to level 2 through level 3.
- If the relaxation from level 3 to level 2 is very fast, then most of the atoms in level 3 will relax down to level 2 rather than to level 1.
- Since the upper pump level 3 is not one of the laser levels, level 3 can be broad level so that a broadband light source can be used efficiently as a pumping

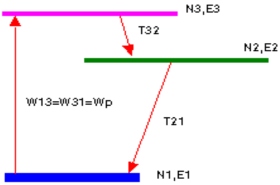

Figure: The energy levels in a three level laser systems

- Let N_1, N_2, and N_3 represent the number of atoms per unit volume in levels $1, 2, 3$, respectively. We assume that only these three levels are populated and that the transitions take place only between these three levels. For such a case, we may write

$$N = N_1 + N_2 + N_3 \tag{1}$$

  where N represents the total number of atoms per unit volume.
- Now the change in the population level 3 is described by

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - N_3 T_{32} \tag{2}$$

  where W_pN_1 represents the number of induced absorptions per unit time per unit volume which results in the $1 \longrightarrow 3$ transition: similarly W_pN_3 represents the number of stimulated emissions per unit time per unit volume associated with $3 \longrightarrow 1$ transition.

- $$T_{32} = A_{32} + S_{32} \tag{3}$$

  where A_{32} represents the Einstein A coefficient connecting levels 3 and 2, and S_{32} represents the nonradiative transition rate from levels 3 to 2.
- The rate of change of the population of level 2

$$\frac{dN_2}{dt} = W_l(N_1 - N_2) + N_3 T_{32} - N_2 T_{21} \tag{4}$$

  - the first term on the right-hand side represents the stimulated transitions between levels 1 and 2
  - the second term represents spontaneous transition from level 3 to level 2 and the third term represents the spontaneous transition from level 2 to level 1.
  - W_l is proportional to the Einstein coefficient B_{21} and to the energy density associated with the lasing transition $2 \longrightarrow 1$.
  - T_{21} represents the net spontaneous relaxation rate from level 2 to level 1. If this transition is predominantly radiative then $T_{21} \approx A_{21}$, where A_{21} represents the Einstein coefficient.

- $$\frac{dN_1}{dt} = W_p(N_3 - N_1) + W_l(N_2 - N_1) + N_2 T_{21} \quad (5)$$

  where the first two terms represent the stimulated transitions between levels 1 and 3 and Levels 1 and 2, respectively, and the last term represents the spontaneous transitions from level 2 to level 1.

- $$\frac{dN_1}{dt} + \frac{dN_2}{dt} + \frac{dN_3}{dt} = 0 \quad (6)$$

  Equations 2, 4 and 5 are referred to as the rate equations and give the rate of change of populations of the three levels in a three-level laser system in terms of W_p and W_l.

At steady state

- $$N_3 = \frac{W_p}{W_p + T_{32}} N_1 \tag{7}$$

- $$N_2 = \left(W_l + \frac{T_{32} W_p}{W_p + T_{32}} \right) \frac{N_1}{W_l + T_{21}} \tag{8}$$

- Therefore the population difference between the levels 2 and 1 is

$$\frac{N_2 - N_1}{N} = \frac{W_p(T_{32} - T_{21}) - T_{32} T_{21}}{3W_p W_l + 2W_p T_{21} + 2T_{32} W_l + T_{32} W_p + T_{32} T_{21}} \tag{9}$$

- In order to obtain population inversion between levels 2 and 1 i.e, $N_2 - N_1$ to be positive, a necessary condition is that $T_{32} > T_{21}$.
- Since the relaxation time of atoms in levels 3 and 2 are inversely proportional to the corresponding relaxation rates, the lifetime of level 3 must be at least smaller than the life time of level 2 for attainment of population inversion.
- In addition, in order to achieve population inversion, a minimum pump power is required.

So

$$W_{pt} = \frac{T_{32} T_{21}}{T_{32} - T_{21}} \tag{10}$$

To obtain population inversion, W_p is required to be greater than $W_p t$. *Book Suggested:Lasers; Theory and Applications, K. Tyagrajan and A. K. Ghatak*