Four-Level Lasers: Rate Equation

Soma Mandal

Department of Physics,
Government Girls' General Degree College

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The Four-Level System

- Consider a four level laser system
- Level 1 is the ground level and levels 2, 3, and 4 are excited levels of the system.
- Atoms from the level 1 are excited by a pump to level 4, from which the atoms decay very rapidly through some nonradiative transition to level 3.
- Level 3 is a metastable level having a long lifetime.

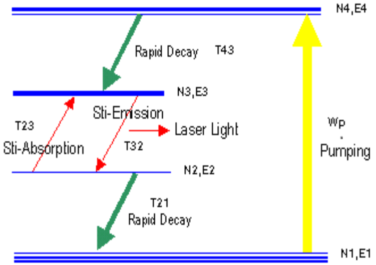

Figure: The energy levels in a four level laser systems.

The Four-Level System

- This level forms the upper laser level and level 2 forms the lower laser level. The lower laser level must have a very short lifetime so that the incoming atoms from level 3 relax down immediately from level 2 to level 1, ready for being pumped to level 4.
- If the rate of relaxation of atoms from level 2 to level 1 is faster than the rate of arrival of atoms into level 2, one can obtain population inversion between level 3 and 2 even for very small pump powers.
- The amount of population inversion between levels 3 and 2 would depend on pumping rate.
- The level 4 could be a collection of large number of levels or a broad level. Then an optical pump emitting radiation over a band of frequencies can efficiently excite atoms from level 1 to level 4. Also level 4 can not be upper laser level because the upper laser level is required to be narrow.
- In addition the lower laser level 2 is required to be sufficiently above the ground level 1 so that at ordinary temperatures the population of level 2 is negligible.
- If the level 2 is not sufficiently above level 1 then the pumping power required for achieving population inversion would be much more. At the same time the population of level 2 may also be lowered by lowering the temperature of the system.

The rate equations

Let N_1, N_2, N_3 and N_4 represents the population per unit volume of the levels 1, 2, 3 and 4 respectively. The change in population of level 4 is given by

$$\frac{dN_4}{dt} = W_p(N_1 - N_4) - T_4 N_4 \tag{1}$$

- $W_p(N_1 - N_4) \rightarrow$ the net rate of stimulated transitions between level 1 and 4 caused by the pump beam
- T_4 is the net relaxation rate from level 4 to the lower levels 3, 2, and 1.

The rate equations

$$T_4 = T_{43} + T_{42} + T_{41} \tag{2}$$

- T's $\rightarrow$ the total relaxation rates (both radiative and nonradiative).
- Also $T_{43} \gg T_{42}$ and T_{41}.

The first term represents the rate at which atoms are being pumped from level 1 to level 4 and the second term represents the rate at which atoms decay from level 4.

Similarly the rate equation for N_3 would be

$$\frac{dN_3}{dt} = T_{43}N_4 + W_l(N_2 - N_3) - T_3N_3 \tag{3}$$

where $T_3 = T_{32} + T_{31}$.

- The first term represents the rate at which atom drop down from level 4 into level 3
- the second term represents the rate of stimulated transitions from level 2 to level 3 due the presence of laser radiation
- the third term represents the rate of loss of atoms from level 3 to level 2 and 1 through spontaneous transitions.

Similarly the rate equations for N_2 and N_1 would be

$$\frac{dN_2}{dt} = T_{42}N_4 + W_l(N_3 - N_2) - T_{21}N_2 + T_{32}N_3 \tag{4}$$

$$\frac{dN_1}{dt} = W_p(N_4 - N_1) + T_{41}N_4 + T_{31}N_3 + T_{21}N_2$$

As the total no. of atoms is constant , we have

$$\frac{dN_1}{dt} + \frac{dN_2}{dt} + \frac{dN_3}{dt} + \frac{dN_4}{dt} = 0 \tag{6}$$

$$N = N_1 + N_2 + N_3 + N_4 \tag{7}$$

At steady state

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dN_3}{dt} = \frac{dN_4}{dt} = 0 \tag{8}$$

Therefore

$$N_4 = \frac{W_p}{2W_p + T_4}(N - N_2 - N_3) \tag{9}$$

Using Eqns. 7, 9 and Eqns. 3 4 and 8, we obtain

$$N_3\left[(W_l+T_3)\left(\frac{2W_p+T_4}{W_p}\right)+T_{43}\right]+N_2\left[T_{43}-\frac{W_l(2W_p+T_4)}{W_p}\right]=T_{43}N \tag{10}$$

$$N_3\left[T_{42}-(T_{32}+W_l)\left(\frac{2W_p+T_4}{W_p}\right)\right]+N_2\left[(T_{21}+W_l)\left(\frac{2W_p+T_4}{W_p}\right)+T_{42}\right]=NT_{42} \tag{11}$$

We now solve the above two equation to obtain the population difference between level 3 and 2, namely, $N_3 - N_2$.
We have

$$\frac{N_3-N_2}{N}=(T_{21}T_{43}-T_3T_{42}-T_{32}T_{43})\times$$
$$\left\{T_{43}(T_{21}+T_{32})+T_3T_{42}+\left(\frac{2W_p+T_4}{W_p}\right)T_3T_{21}+W_l\left[2(T_{42}+T_{43})+\left(\frac{2W_p+T_4}{W_p}\right)(T_{31}+T_{21})\right]\right\}^{-1} \tag{12}$$

This equation gives the steady state population difference as a function of the pump power, the laser power, and the lifetimes of the various states involved in the four level system.

In most four-level systems, the atom from level 4 relax primarily to level 3 and hence $T_{42}, T_{41} << T_{43}$. Also the atoms from level 3 relax primarily from level 2, i.e., $T_3 \approx T_{32}$. Under these approximations, we see that in order to obtain a population inversion, we must have

$$T_{21} > T_{32} \tag{13}$$

Under such a condition, the creation of population inversion betweens level 3 and 2 is independent of the pumping power but the magnitude of the population inversion depend on W_p.
At and below threshold conditions, $W_l \approx 0$ so the approximate equation for the population difference is given by

$$\frac{N_3 - N_2}{N} = \frac{(1-\beta) W_p T_{43}/T_4 T_3}{1 + [(1+\beta) + 2T_3/T_{43}]\, W_p T_{43}/T_4 T_3} \tag{14}$$

where

$$\beta = \frac{T_{32}}{T_{21}} + \frac{T_3 T_{42}}{T_{21} T_{43}} \tag{15}$$

For good laser action one must have $T_3 << T_{43}$ and $T_{21} >> T_{32}$ so that $\beta \to 0$. Also $T_4 \approx T_{43}$. Therefore

$$\frac{N_3 - N_2}{N} = \frac{W_p / T_3}{1 + W_p / T_3} \tag{16}$$

If we compare the steady state population difference between three level and four level laser system we can see that the inversion can be much more easily obtained in a four level scheme as compared to a three level system.

Book Suggested:Lasers; Theory and Applications, K. Tyagrajan and A. K. Ghatak