Nuclear Models

Liquid Drop Model

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What have we learnt about the Nucleus?

- The nuclear density is roughly constant for all nuclei.
- Nuclei are positively charged, and the nuclear charge density is also roughly constant.
- The strong force is attractive only at short range and is repulsive at very short range.

Introduction

- In order to understand the observed properties of the nucleus of an atom it is necessary to have an adequate knowledge about the nature of the internucleon interaction. But none of the proposed theories gives us a full understanding of the nature of the internucleon interaction. Even if the the nature of internucleon interaction were known, it would have been extremely difficult to develop a satisfactory theory of the structure of the nucleus made up of a large number of protons and neutrons, since it is almost impossible to solve the Schrödinger equation exactly for such a many body system.
- Even if one uses nucleons as a fundamental degrees of freedom, the system is too small for statistical approach too. Thus the problem is complicated and this is probably one of the main explanation for the existence of large number of theoretical approaches that have been (and are) used. Therefore we make use of models and use simple analogies.

The Liquid Drop Model

A simple approach to modelling the atomic nucleus

- The scattering experiment suggested that nuclei have approximately constant density.
- We were then able to calculate the nuclear radius assuming a uniform sphere. A drop of uniform liquid has the same property.
- The nuclear force is short-range, but does not allow for compression of nuclear matter. Molecules in a liquid drop have the same basic properties.
- The nucleus is a positively charged object. For our purposes here, we can assume our liquid drop also has a uniform positive charge.
- We have been assuming spherical nuclei so far, but when additional energy is introduced into the system, nuclei can change their shape. A drop of liquid has the same property, and when other forces are present, it can deviate from a spherical shape.

Bethe-Weisaäcker formula:

- This semiemperical formula for the nuclear mass gives a connection between the nuclear matter with experimental information and is based on the liquid drop model of the nucleus.
- The mass of a nucleus having Z protons and N neutrons is given by

$$M = ZM_H + NM_n - E_B$$

where $M_H \rightarrow$ mass of hydrogen ion
$M_n \rightarrow$ mass of a neutron
$E_B \rightarrow$ binding energy, defined as the minimum work nesessary to dissociate a nucleus into its component nucleons.
E_B is made up of a number of terms as discussed below, where we have assumed the nucleus to behave like a liquid drop of an incompressible liquid of constant density.

The Liquid Drop Model

The Volume Energy Term

- The volume energy term due to exchange forces gives the largest contribution to E_B. The term can be identified as the first term in binding energy and may be expressed as

$$E_0 = a_v A$$

, where a_v is a constant which can be determined from the known values of masses; E_0 is called volume energy, since just like nuclear volume, this term is proportional to mass no A.

Surface Energy Term

- The nucleus of an atom has some nucleons on its surface. These nucleons do not have as many neighbours as the nucleons in the interior. The exchange forces in the interior will get saturated but those on the surface will remain unsaturated. The volume energy E_0 will have a correction term E_1 representing the nucleoons on the surface.
  Now, the radius of the nucleus is given by $R = r_0 A^{1/3}$, where r_0 is a constant. Therefore nuclear surface area $= 4\pi R^2 = 4\pi {r_0}^2 A^{2/3}$. Hence the number of nucleons exposed to the surface will be proportional to $A^{2/3}$. Thus the negative correction term will be given by

$$E_1 = -a_s A^{2/3}$$

, where a_s is a constant.

Coulomb Energy Term

- Coulomb repulsive forces are produced due to the mutual repulsion of positively charged protons in the nucleus. The protonic charge Ze is evenly distributed in the entire volume of a nucleus. Just like the surface energy, the coulombian energy E_2 will have a $-ve$ contribution and is given by

$$\begin{aligned} E_2 &= -\frac{3}{5}\frac{Z^2e^2}{R} \\ &= -\frac{3}{5}\left(\frac{Z^2e^2}{r_0A^{1/3}}\right) \\ &= -a_c\frac{Z^2}{A^{1/3}} \end{aligned} \tag{1}$$

$$a_c = \frac{3}{5}\frac{e^2}{r_0} = constant$$

The Liquid Drop Model

Asymmetric Energy Term

- In heavy nuclei the number of neutrons exceeds that of protons by a large factor. Only in a few light nuclei $Z = N$. It is essential to add more neutrons to provide stability to the nuclei against the Coulomb's repulsion due to a large number of protons in a heavier nuclei. The energy states of the individual nucleons in the nucleus are however quantised and the nucleons are arranged according to Pauli's exclusion principle. If we put Z protons in certain low energy states, an equal no. of neutrons can be accommodated in theses states along with the protons. The $(N - Z)$ excess neutrons will therefore go to the higher unoccupied quantum states which are the states of larger kinetic energy. Since the binding energy is equal to $(P.E - K.E)$, these $(N - Z)$ excess neutrons will have a much smaller binding. Thus, these $(N - Z)$ excess neutrons will produce a deficit in binding energy of a fraction of the nuclear volume.

The Liquid Drop Model

Asymmetric Energy Term

- The fraction of nuclear volume affected per nucleon is $\left(\frac{N-Z}{A}\right)$. Therefore total deficit in binding energy

$$\begin{aligned} \propto \left(\frac{N-Z}{A}\right)(N-Z) &= -a_a \frac{(N-Z)^2}{A} \\ &= -a_a \frac{(A-2Z)^2}{A} \end{aligned}$$

, where a_a is a constant. Hence the assymmetric energy is

$$E_3 = -a_a \frac{(A-2Z)^2}{A} \tag{2}$$

- The nuclear binding energy is somewhat affected by the spin angular momentum of the nucleons. For nuclei of even A, Z and N may be both even or both odd. If these numbers are even, the nucleus may be grouped into stable pair, with sign opposed and the nucleus will be correspondingly more stable than in the case of Z and N being odd. The pairing energy contribution is zero for odd A. The pairing energy is actually represented by
  $E_\delta = +33A^{-3/4}$ Mev for even (Z)-even (N) nuclei;
  $= 0$ for even-odd/odd-even nuclei;
  $= -33A^{-3/4}$ Mev for odd-odd nuclei.

Bethe-Weisaӓcker semi emperical mass formula:

$$E_B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + E_\delta$$

- Thus, the Bethe-Weisaӓcker semi emperical mass formula becomes,

$$\begin{aligned} M(Z,A) &= ZM_H + NM_n - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{A^2 - 4AZ + 4Z^2}{A} - E_\delta \\ &= \left[(M_n - a_v + \frac{a_s}{A^{1/3}} + a_a)A - E_\delta\right] + \left[M_H - M_n - 4a_a\right] Z + \left(\frac{a_c}{A^{1/3}} + \frac{4a_a}{A}\right) Z^2 \\ &= \alpha A + \beta Z + \gamma Z^2 - E_\delta \end{aligned}$$

Bethe-Weisaӓcker semi emperical mass formula:

$$\begin{aligned} M(Z,A) &= ZM_H + NM_n - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{A^2 - 4AZ + 4Z^2}{A} - E_\delta \\ &= \left[(M_n - a_v + \frac{a_s}{A^{1/3}} + a_a)A - E_\delta\right] + [M_H - M_n - 4a_a]\,Z + \left(\frac{a_c}{A^{1/3}} + \frac{4a_a}{A}\right) Z^2 \\ &= \alpha A + \beta Z + \gamma Z^2 - E_\delta \end{aligned}$$

$$\alpha = M_n - a_v + \frac{a_s}{A^{1/3}} + a_a$$

$$\beta = M_H - M_n - 4a_a$$

$$\gamma = \frac{a_c}{A^{1/3}} + \frac{4a_a}{A}$$

Thus the relation between $M(Z,A)$ and Z is a parabolic one.

Books Suggested:

- Concept of Modern Physics, Arthur Beiser
- Atomic and Nuclear Physics, Vol II; S. N. Ghosal
- Nuclear Physics, Theory and Experiment; R. R. Roy & B. P. Nigam
- Atomic and Nuclear Physics, An Introduction; T.A. Littlefield & N. Thorley