Review of SHM

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Review of SHM, damped and forced vibrations: amplitude and velocity resonance. Fourier's Theorem amd its application for some waveforms e.g., Saw tooth wave, triangular wave, square wave. Intensity and loudness of sound. Intensity levels, Decibels.

1 Simple Harmonic Oscillation

The simplest of all periodic motions is simple harmonic motion. In this type of motion a system vibrate about their mean positon of rest and the displacement is a circular function of time.

If the force acting on the oscillating body is always in the direction opposite to the displacement of the body from the equilibrium or the mean position and its magnitude is proportional to the magnitufe of the displacement, the body is said to be executing simple harmonic motion (SHM).

If $\vec{F}$ is the restoring force which tries to restore the particle at the mean position we can write $\vec{F} \propto -\vec{x}$.
$\therefore F = -kx$

The differential form

$$m\frac{d^2x}{dt^2} = -kx \tag{1}$$

or

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{2}$$

The above equation is the differential equation of SHM, where ω is the angular frequency.
Let $x = e^{\alpha t}$ be a trial solution. Auxilliary equation is $\alpha^2 + \omega^2 = 0$. $\therefore \alpha = \pm i\omega$
The general solution is

$$x = A\cos\omega t + B\sin\omega t \tag{3}$$

,where A and B are two arbitrary constant.

Let $t = 0, x = 0 \therefore A = 0$.
Therefore $x = B\sin\omega t$; x is maximum when $\sin\omega t = 1$; $x = B = a$=amplitude.

$$\therefore x = a\sin\omega t \tag{4}$$

At $t = 0, x = a$ and $\frac{dx}{dt} = 0$
$a = A$

$$\therefore x = a\cos\omega t \tag{5}$$

$\frac{dx}{dt} = -A\omega \sin \omega t.$
Hence $\frac{dx}{dt}$ at t=0 is zero. When the particle start oscillating from the amplitudinal position then the function is cosine function and when the particle statrs from the mean position of equlibrium the function is sine function.

The time dependence can also be written in the form

$$x \equiv A\cos(\omega t + \phi) \equiv Pe^{i\omega t} + Qe^{-i\omega t} \tag{6}$$

The SHM equation has a first integral which can be obtained by

$$\frac{d^2x}{dt^2} \equiv \frac{dv}{dt} = \frac{dx}{dt}\frac{dv}{dx}$$

or equivalently, by multiplying the equation 1 by $\dot{x}$

$$m\dot{x}\ddot{x} = -k\dot{x}x$$

and integrating

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = E \tag{7}$$

where E is constan of integration. The first term of Equaion 7 has the obvious interpretation of the kinetic eergy of the particle. It would not be surprising, therefore, if the second term could be interpreted as its potential energy, and this is easily demonstrated. The workdone on the particle to move it from the equilibrium point to displacement x is

$$\int (-F)dx = \int (kx)dx = \frac{1}{2}kx^2$$

as expeced.

We can check that the solution $x = A\cos(\omega t + \phi)$ satisfies Equation 7

$$x = A\cos(\omega t + \phi) \longrightarrow \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \phi) + \frac{1}{2}kA^2\cos^2(\omega t + \phi) = \frac{1}{2}kA^2$$

using $k = m\omega^2$. Thus $E = \frac{1}{2}kA^2$ which is constant, as required.

The average kinetic and potential energy may also be calculated in the following way

Average kinetic energy

$$\frac{1}{T}\int_0^T \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 dt$$

Average potential energy over an oscillator

$$\frac{1}{T}\int_0^T \frac{kx^2}{2}dt$$

1.1 Assignment I

1. A paticle of mass 0.2 kg undergoes SHM accordng to the equation: $x(t) = 3\sin(\pi t + \pi/4)$. [t is in s and x in m].
(a) What is the amplitude of oscillation?
(b) What is the time period of oscillation?
(c) What is the initial value of x?
(d) What is the initial velocity when the SHM starts?
(e) A what instants is the particle's energy purely kinetic?

2. A particle moves on the $x-$ axis according to the equation $x = A + B\sin\omega t$. Is the motion SHM? Ifyes, what is the amplitude?

3. What would be the distance moved by a particle in SHM in one time period ?

4. A particle moves with SHM of amplitude 20 cm and period 4 sec. The displacement at $t = 0$ is $+20$ cm. Find the position of the particle at $t = 0.5$ sec.