matrices

Dr. Soma Mandal

Department of Physics

Government girls' general degree college

Syllabus

3. Matrices **15 Lectures**

(a) Addition and Multiplication of Matrices. Null Matrices. Diagonal, Scalar and Unit Matrices. Transpose of a Matrix. Symmetric and Skew-Symmetric Matrices. Conjugate of a Matrix. Hermitian and Skew- Hermitian Matrices. Singular and Non-Singular matrices. Orthogonal and Unitary Matrices. Trace of a Matrix.

(b) Eigen-values and Eigenvectors (Degenerate and non-degenerate). Cayley-Hamiliton Theorem. Diagonalization of Matrices. Solutions of Coupled Linear Ordinary homogeneous Differential Equations. Functions of a Matrix.

WHAT DO YOU MEAN BY MATRICES?

- Let us consider a set of simultaneous equations,

$$x + 2y + 3z + 5t = 0$$
$$4x + 2y + 5z + 7t = 0$$
$$3x + 4y + 2z + 6t = 0$$

- A= $\begin{bmatrix} 1 & 2 & 3 & 5 \\ 4 & 2 & 5 & 7 \\ 3 & 4 & 2 & 6 \end{bmatrix}$

- A system of numbers, arranged in a rectangular array in rows or columns and bounded by the brackets, is called a matrix.

VARIOUS TYPES OF MATRICES

- Row Matrix [2,7,3,9]
- Column Matrix $\begin{bmatrix} 2 \\ 8 \\ 9 \end{bmatrix}$
- Null/Zero Matrix : A matrix in which each entry is zero is called a zero matrix. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- Square Matrix: : A matrix having the number of rows is equal to the number of columns is called a square matrix. $\begin{bmatrix} -1 & 2 \\ 4 & 9 \end{bmatrix}$, $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 3 \\ 4 & 2 & 0 \end{bmatrix}$
- Diagonal Matrix: A square matrix is said to be diagonal matrix if the non-zero entries appear only on the principal diagonal. $\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$
- A diagonal matrix D of order n with the diagonal entries $d_1, d_2, d_3 \ldots \ldots \ldots d_n$ is denoted by $D = diag(d_1, \ldots \ldots, d_n)$. If $d_i = d$ for all $i = 1, 2, \ldots . n$ then the diagonal matrix D is called a scalar matrix.

VARIOUS TYPES OF MATRICES

- Unit or Identity Matrix: A square matrix $A = [a_{ij}]$ with $a_{ij} \begin{cases} 1 \ if \ i = j \\ 0 \ if \ i \neq j \end{cases}$ is called the identity matrix .

  Example: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- A square matrix is set to be upper triangular if $a_{ij} = 0 \ for \ i > j$. Example: $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 4 & 3 \\ 0 & 0 & 6 \end{bmatrix}$

- A square matrix is set to be lower triangular if $a_{ij} = 0 \ for \ i < j$. Example: $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 0 \\ 2 & 3 & 6 \end{bmatrix}$

- A square matrix is said to be Triangular Matrix if it is an upper or lower triangular matrix.