Matrices: operation on matrices

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OPERATION ON MATRICES

- Transpose of a matrix
- The transpose of a $\mathrm{m} \times n$ matrix $A = [a_{ij}]$ is defined as the $n \times m$ matrix $B = [b_{ij}]$ with $b_{ij} = a_{ij}$ for $1 \le i \le m$ and $1 \le j \le n$. The transpose of A is denoted by A^T.

If $A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix}$, then $A^T = \begin{bmatrix} 1 & 0 \\ 4 & 1 \\ 5 & 2 \end{bmatrix}$

Thus the transpose of a row vector is a column vector and vice-versa.

- For any matrix A, we have ${A^T}^T = A$.
- Let $A = [a_{ij}]$, $A^T = [b_{ij}]$ and ${A^T}^T = [c_{ij}]$.

Then the definition of transpose gives $c_{ij} = b_{ji} = a_{ij}$ for all i, j and the result follows.

SYMMETRIC, SKEW-SYMMETRIC AND ORTHOGONAL MATRIX

- Symmetric matrix : A matrix A is called symmetric if $A^T = A$

  A= $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$, B= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & -1 \\ 3 & -1 & 4 \end{bmatrix}$ are symmetric matrices.

- Skew Symmetric matrix: A matrix C is called skew symmetric if $A^T = -A$

  If C= $\begin{bmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$, C is a skew symmetric matrix.

- A matrix is said to be orthogonal if $AA^T = A^TA = I$.

- A= $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \end{bmatrix}$ is an orthogonal matrix.

ADDITION OF MATRICES

- Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two $m \times n$ matrices. Then the sum $A + B$ is defined to be the matrix $C = [c_{ij}]$ with $c_{ij} = a_{ij} + b_{ij}$. Matrices must have the same dimensions.

  A= $\begin{bmatrix} 4 & 2 & 5 \\ 1 & 3 & -6 \end{bmatrix}$ B= $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$ A+B= $\begin{bmatrix} 5 & 2 & 7 \\ 4 & 4 & -2 \end{bmatrix}$

- Subtraction is identical to addition. $\begin{bmatrix} 9 & 4 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & -5 \end{bmatrix}$
- Properties of matrix addition:
- Communicative law of addition A+B=B+A
- Associative law of addition A+(B+C)= (A+B)+C
- Exercise:

  Suppose A+B=A. Then show that B=0.

  Suppose A+B=0. Then show that B=(-1)A=$[-a_{ij}]$.

MATRIX MULTIPLICATION

- Multiplying a scalar to a matrix

  Let $A = [a_{ij}]$ be an $m \times n$ matrix, for any element k, $kA = [ka_{ij}]$.

  If $A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix}$ and $k = 3$; then 3A=$\begin{bmatrix} 3 & 12 & 15 \\ 0 & 3 & 6 \end{bmatrix}$.

- Matrix multiplication/Product:

  Let $A = [a_{ij}]$ be an $m \times n$ matrix and $B = [b_{ij}]$ be an $n \times r$ matrix. The product AB is a matrix $C = c_{ij}$ of order $m \times r$ with

  $c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$=$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$.

  The product AB is defined if and only if

  THE NUMBER OF COLUMNS OF A= THE NUMBER OF ROWS OF B

- To multiply two matrices

1) Multiply each element in a given row by each element in a given column.
2) Sum up their products.

MATRIX MULTIPLICATION

- *Example:*

- $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$=$\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$=$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$
- $\begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$=[$(3 \times 1) + (2 \times 3)$ $(3 \times 2) + (2 \times 4)$] = $\begin{bmatrix} 9 & 14 \end{bmatrix}$

- **Properties of matrix multiplication**

- Matrix multiplication is not commutative. $AB \neq BA$.

  Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; B=$\begin{bmatrix} 0 & -1 \\ 6 & 7 \end{bmatrix}$

  $AB = \begin{bmatrix} 12 & 13 \\ 24 & 25 \end{bmatrix}$ but $BA = \begin{bmatrix} -3 & -4 \\ 27 & 40 \end{bmatrix}$
- Associative law: Matrix multiplication is associative $(AB)C = A(BC) = ABC$
- Distributive law: Matrix multiplication is distributive with respect to addition A(B+C)=AB+AC
- For any k, $(kA)B = \mathrm{k(AB)} = \mathrm{A(kB)}$.
- If A is an $n \times n$ matrix then $AI_n = I_nA = A$.

EXERCISE:

- Let A and B two matrices. If the matrix addition A+B is defined, then prove that $(A+B)^T = A^T + B^T$. Also if the matrix product AB is defined then prove that $(AB)^T = B^T A^T$.
- Let $A = [a_1, a_2, \ldots. a_n]$ and $B = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_n \end{bmatrix}$. Compute the matrix product AB and BA.