2021

ECONOMICS — HONOURS

Second Paper

(Group - B)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

SECTION – A

Answer ***any five*** questions.

1. (a) Enumerate all the subsets of the set $A = \{1, 3, 5, 7\}$. How many subsets are there all together?

(b) Given $A = \begin{bmatrix} -1 & 5 & 7 \\ 0 & -2 & 4 \end{bmatrix}$, show that $AI = IA = A$. Indicate the dimension of identity matrix used in each case. 2+2

2. State the Euler's theorem and verify it for the function $Y = A \cdot x_1^{\alpha} . x_2^{\beta}$ where $A, x_1, x_2 > 0$, $0 < \alpha, \beta < 1$ and $\alpha + \beta = 1$. 2+2

3. Solve the following system of linear equations applying the Cramer's rule :

$$4x_1 + 3x_2 - 2x_3 = 7$$
$$x_1 + x_2 = 5$$
$$3x_1 + x_3 = 4$$

4

4. Suppose that the profit (π) of a firm depends upon research (R) and advertisement (A) expenditures in the following way :

$$\pi = -R^2 - A^2 + 22R + 18A - 102$$

Find out the optimum research and advertisement expenditures of the firm for profit maximization. 4

5. Solve the following linear differential equation and verify the solution :

$$\frac{dy}{dt} + 5y = 15; \; y(0) = 1$$

3+1

6. Use first derivative and second derivative of the following function to sketch the graph of the function $f(x) = x^3 - 3x$. 4

7. Find out the mixed strategy solution for the following zero-sum game :

	Player II	
		Left	Right
Player I	Up	4	–2
	Down	–5	4

4

8. Given,

$$A=\begin{bmatrix}1 & 2\\ 3 & 6\end{bmatrix},\ B=\begin{bmatrix}3 & -8\\ 2 & 3\end{bmatrix},\ C=\begin{bmatrix}5 & 2\\ 1 & -2\end{bmatrix},\ D=\begin{bmatrix}\frac{1}{6} & \frac{1}{6}\\ \frac{1}{12} & \frac{-5}{12}\end{bmatrix}$$

(a) Verify that $AB = AC$ even though $B \neq C$.

(b) Are C and D inverse to each other ? 2+2

SECTION -B

Answer ***any five*** questions.

9. Consider the function :

$$\begin{aligned} f(x) &= 1 + x && \text{if } x < 0 \\ &= x^2 + x + 1 && \text{if } x > 0 \\ &= 1 && \text{if } x = 0 \end{aligned}$$

(a) Sketch the graph of the function.

(b) Is $f(x)$ continuous? Is it smooth?

(c) Using the definition of continuity, check whether $f(x)$ is continuous at $x = 0$ or not. 2+1+1+2

10. Consider $z = e^{x^2y+xy^2}$

(a) Is the function z homogeneous?

(b) Check the homogeneity of $F(z) = \ln z$.

(c) Is the function z homothetic? 2+1+3

11. Determine the values of the constants a, b and c such that the function $f(x, y) = ax^2y + bxy + 2xy^2 + c$ has a local minimum at the point $\left(\frac{2}{3}, \frac{1}{3}\right)$ with local minimum value $= -\frac{1}{9}$. 6

12. Given the following input-output table for a three-industry model—

Industry	I1	I2	I3
I1	0·3	0·2	0·2
I2	0·2	0·1	0·5
I3	0·2	0·4	0·2
Labour	0·4	0·3	0·1

(a) Check whether the model satisfies the Hawkins-Simon conditions.

(b) If the optimum output levels for I1, I2 and I3 are 241 units, 215 units and 230 units respectively and the total labour supply $\bar{L} = 200$, will there be any unemployment in the economy? 4+2

13. Consider the production function $Q = AK^{\alpha}L^{\beta}$, $A, \alpha, \beta > 0$.

(a) Show that the function has the property of increasing marginal productivity of capital and labour if $\alpha > 1$ and $\beta > 1$.

(b) What will be the shape of the isoquant (level curve of the production function)? 3+3

14. Maximise $U(x, y) = x^{\alpha} y^{\beta}$ $(x, y > 0, \alpha, \beta > 0)$

Subject to $M = xp_x + yp_y$ $(M, p_x, p_y > 0)$

(a) Find the demand functions of x and y by using the logarithmic transformation of the given function. [Assume that the second order sufficient conditions are satisfied].

(b) Explain the justification of using the above transformation.

(c) Show that the demand functions of x and y are homogeneous of degree zero in money income and absolute prices. 3+2+1

15. Maximise $\pi = 3y_1 + 4y_2 + 3y_3$

Subject to $y_1 + y_2 + 3y_3 \leq 12$

$2y_1 + 4y_2 + y_3 \leq 42$

$y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0$

(a) Write down the dual of the above primal problem.

(b) Solve the dual problem graphically. 2+4

Please Turn Over

16. Consider the following game—

A \ B	B_1	B_2
A_1	6, 6	4, 6
A_2	6, 4	0, 0

(a) What do you mean by a 'dominant strategy'? Obtain the dominant strategies for player *A* and *B*.

(b) Define a Nash Equilibrium. Does the game have any pure strategy Nash Equilibrium? If so, what are they? 1+2+1+2